Updates

1. Exam I will be administered on Monday, March 16. You will need to bring a **SMALL** blue book. It will cover up through the maximum principle, so you should complete the first two problems on the homework by then.

Homework Set 4 (Revised)

Read sections 1.3, 1.4, 8.1–8.3.

The Heat Equation

1. Consider the following heat equation:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \qquad x \in [0,1], \quad t > 0, \tag{4.1a}$$

$$T(x,0) = 0, \qquad \frac{\partial T}{\partial x}(0,t) = 0, \qquad T(1,t) = e^{-3t}. \tag{4.1b}$$

(a) (7 points) Introduce a new variable $\phi(x,t)$ such that the problem (4.1) has homogeneous boundary conditions for ϕ, and write the new problem for ϕ which results.

(b) (13 points) Show that the solution of (4.1) is given by

$$T(x,t) = h(t) + \sum_{n=0}^{\infty} \frac{f_n}{\lambda_n^2 - 3}\left[3h(t) - \lambda_n^2 e^{-\lambda_n^2 t}\right]\cos(\lambda_n x),$$

and find the values of $h(t)$, f_n, and λ_n.

The Maximum Principle

2. (7 points) Let ϕ_1 and ϕ_2 satisfy the heat equation on $x \in [0,1]$, subject to the following boundary conditions:

$$\phi_1(0,t) = f_1(t) < f_2(t) = \phi_2(0,t), \quad \phi_1(1,t) = g_1(t) < g_2(t) = \phi_2(1,t), \tag{4.2a}$$

$$\phi_1(x,0) = h_1(x) < h_2(x) = \phi_2(x,0). \tag{4.2b}$$

Use the maximum/minimum principle to show that $\phi_1 < \phi_2$ for all x and t.

Classification

3. (13 points) Consider the following equation:

$$x\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 4x = 0,$$

which holds in the entire x-y plane. Classify it and transform it into canonical form. What happens when $x = 0$?