Homework Set 3

Read sections 3.1–3.4.

Trigonometric Fourier Series

1.
(a) (8 points) Find the Fourier series of

$$f(x) = \sin^2 \nu x + \sin 2\nu x, \quad x \in [-\pi, \pi], \quad 2\nu \text{ not an integer.}$$

(b) (4 points) By substituting in a particular value of x, use your answer to (a) to show that

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n^2 - 4\nu^2)} = \frac{1}{8\nu^2} - \frac{\pi}{4\nu \sin 2\nu\pi}.$$

2.
(a) (8 points) Find the Fourier series of

$$f(x) = \begin{cases} 2x, & -\pi < x \le 0, \\ x, & 0 \le x < \pi. \end{cases}$$

(b) (5 points) Use the non-orthonormal version of Parseval's Theorem and your answer to (a) to show that

$$\sum_{n=1}^{\infty} \frac{2[1 - (-1)^n]}{n^4 \pi} + \frac{9\pi}{n^2} = \frac{37\pi^3}{24}.$$

Hint: Recall that since the Fourier series has two different eigenfunctions, the correct version of the sum side of Parseval's Theorem is

$$\left(\frac{a_0}{2}\right)^2 ||1||^2 + \sum_{n=1}^{\infty} a_n^2 ||\cos nx||^2 + b_n^2 ||\sin nx||^2.$$

Even/Odd Functions

3. Consider the following function:

$$f(x) = \begin{cases} \pi/2, & 0 \le x < \pi/2, \\ \pi - x, & \pi/2 < x \le \pi. \end{cases}$$

(a) (7 points) Sketch the odd extension of f and show that its sine series is given by

$$f(x) = \sum_{m=1}^{\infty} \frac{\sin 2mx}{2m} + \frac{1}{2m-1}\left[1 - \frac{2(-1)^m}{(2m-1)\pi}\right]\sin((2m-1)x) \qquad (3.1a)$$

(b) (7 points) Sketch the even extension of f and show that its cosine series is given by

$$f(x) = \frac{3\pi}{8} + \sum_{m=1}^{\infty} \frac{2\cos(2m-1)x}{(2m-1)^2\pi} + \frac{(-1)^m - 1}{2m^2\pi}\cos(2mx). \qquad (3.1b)$$