Updates

1. The mathematical sciences department will hold its annual Undergraduate Research Mixer on Thursday, Feb. 19 from 1–3 in Trabant 209/211. Faculty from the department will be there to talk about summer undergraduate research opportunities they have available.

Homework Set 2 (Second Revision)

Read sections 1.1, 2.1-2.3.

The Heat Equation

1. (3 points) Suppose that in the derivation of the heat equation (without sources), c_p, ρ, and k all depend on t. Write the new form of the heat equation that results.
2. The heat equation is in *steady state* when $\partial T/\partial t = 0$. At this point, the system is in equilibrium.

   (a) (3 points) Show that in steady state, the heat flux is constant.

   Consider a slender rod, insulated except at the edges, which is placed through an exterior wall ($x = 0$). The exterior end ($x = -1$) is exposed to the outside air at a temperature of $10°$, while the interior end ($x = 1$) is exposed to the inside air at a temperature of $72°$.

   (b) (3 points) Find the steady-state temperature of the bar.
   (c) (2 points) What is the temperature of the rod at the wall?

Separation of Variables

3. (8 points) page 35, exercise 2.2.5
4. (6 points) For the equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right),$$

   find the ordinary differential equations implied by separation of variables.

5. (9 points) Use separation of variables to find the particular solutions (eigenfunctions) of the following problem, which is assumed to have a finite solution:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0, \qquad -1 < x < 1, \quad y < 0$$
$$f(0, y) = \frac{\partial f}{\partial y}(1, y) = 0.$$

Orthogonality

6. (6 points) page 52, exercise 2.3.5