MATH 535-011 Introduction to Partial Differential Equations
Prof. D. A. Edwards Distributed: May 13, 2015

# Updates

1. This is not a homework assignment that I am collecting, since there isn't any time left in the term. However, these are the questions I **would** assign if we had time, so they are indicative of the material I feel is important.
2. Please remember to fill out your online course evaluations at `http://www.udel.edu/udsis-students/courseevaluations.html`.
3. Teacher evaluations may also be given at `http://www.ratemyprofessors.com/ShowRatings.jsp?tid=1621`.
4. There will be an informal review session from 10–12 on Friday, May 22, in EWG 210.
5. The final exam will be from 7–10 pm on Tuesday, May 26, in the regular classroom.

# Supplemental Study Material

Read sections 10.5, 12.3.2, 12.3.3, 12.4.

## Sine and Cosine Transforms

1. Calculate the Fourier sine transform of

$$f(x) = \frac{\sin x}{1 + x^2}.$$

2.

(a) Show that

$$\mathcal{F}_c(f''(x)) = -\frac{f'(0)}{\pi} - k^2 \hat{f}_c. \tag{S.1}$$

(b) Calculate $\hat{f}_c$, where

$$f(x) = e^{-bx^2/2},$$

and use your result to show that

$$\hat{g}_c = \frac{b - k^2}{\sqrt{2\pi b}} e^{-k^2/2b}, \qquad g(x) = b^2 x^2 e^{-bx^2/2}. \tag{S.2}$$

(c) Use your answers to (a) and (b) to show that the solution of

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \quad x > 0,\ t > 0; \qquad \frac{\partial T}{\partial x}(0,t) = 0, \quad T(x,0) = x^2 e^{-x^2/2} \tag{S.3}$$

is given by

$$T(x,t) = \frac{1}{(2t+1)^{3/2}} \left( \frac{x^2}{2t+1} + 2t \right) \exp\left( -\frac{x^2}{2(1+2t)} \right).$$

## d'Alembert's Solution

3. Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

on the fully-infinite interval subject to

$$u(x,0) = \sin(x^2), \qquad \frac{\partial u}{\partial t}(x,0) = x\cos(x^2).$$

Read sections 14.1, 14.7.3.

## Similarity Solutions

4. Consider the nonlinear partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left( u \frac{\partial u}{\partial x} \right). \tag{S.4a}$$

Assume a similarity solution of the form

$$u(x,t) = t^b f(\xi), \qquad \xi = x t^b \tag{S.4b}$$

and show, for properly chosen $b$, that the ODE for $f$ that results is

$$3 \frac{d}{d\xi}\left( f \frac{df}{d\xi} \right) + \xi \frac{df}{d\xi} + f = 0.$$

Do **NOT** attempt to solve this equation.