MATH 535-011 Introduction to Partial Differential Equations
Prof. D. A. Edwards Due: Feb. 18, 2015

# Homework Set 1 (Second Revision)

Note: On this homework assignment, five problems will be graded at 6 points each, and five will be checked in at 2 points each.

## ODEs

1. Find the general solution of

$$(\sin x)y' - (\cos x)y = \sin 2x.$$

2. Find the general solution of

$$y'' - 3y' + 2y = -65\cos 2x.$$

3. Find the solution of

$$y'' + 2y' + 5y = -13e^{-4x}, \qquad y(0) = 5, \quad y'(0) = 4.$$

4. Consider the following problem:

$$y'' + \lambda^2 y = 0, \qquad y'(0) = 0, \quad y(1) = \epsilon; \qquad \lambda > 0. \tag{1.1}$$

   (a) Find the solution of (1.1).
   (b) As $\epsilon \to 0$, is there a way to choose $\lambda$ so you continue to get a nonzero solution?

5. Find the general solution of

$$x^3 y^{(3)} + 5x^2 y'' + 2xy' = 0.$$

## Vector Calculus

6. Find the gradient and Laplacian of

$$f(x, y, z) = x^2(\cos y)e^{-z}.$$

7. For the two examples below, calculate the terms on each side of the Divergence Theorem and verify that they are equal.

$$\begin{aligned} &\text{(a)}: \qquad f(x,y,z) = x^2(\cos y)e^{-z}, \qquad V = [0,1]\times[0,\pi]\times[0,1]. \\ &\text{(b)}: \qquad g(x,y,z) = \exp\left(-x^2-y^2-z^2\right), \qquad V: x^2+y^2+z^2 \le 1. \end{aligned}$$

(*Hints for (b): What coordinate system should you use? Also, using integration by parts on one part of one integral will induce a useful cancellation. The answer is* $-8\pi e^{-1}$.)

## Orthogonality

8. Find a unit vector in the direction of $\mathbf{v} = (-4, 2, -2, 1)$.
9. Find two vectors which are orthogonal to each other and also orthogonal to $\mathbf{w} = (3, -7, -5, 2)$.

## Integration by Parts

10. Let $y$ be *any* nonconstant function such that $y'(a) = y'(b) = 0$, where $a < b$. Show that

$$\int_a^b yy''\,dx < 0.$$