Comparison of Fourier and Laplace Series

In class, we showed that two representations of the solution to the following heat equation:

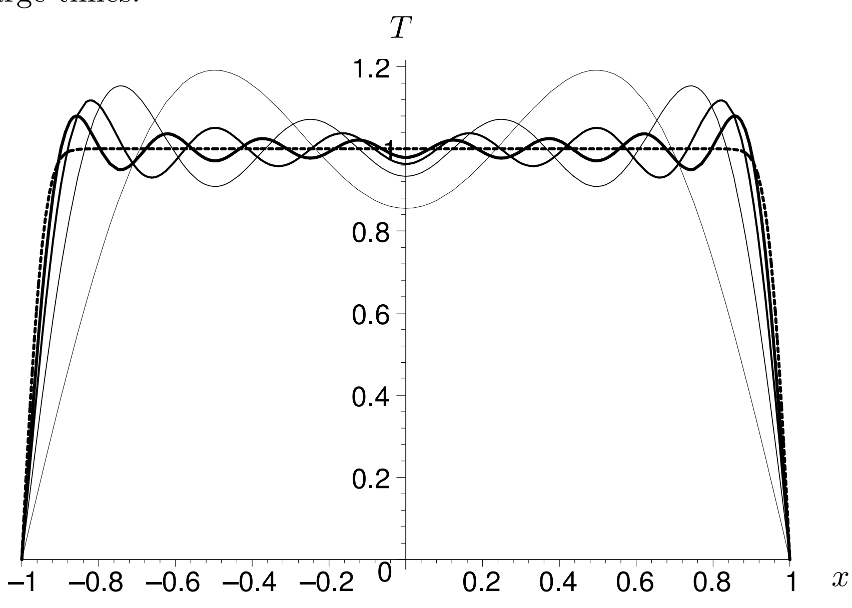
$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \qquad T(-1,t) = T(1,t) = 0, \quad T(x,0) = 1$$

were given by the limits as $N \to \infty$ of the following sums:

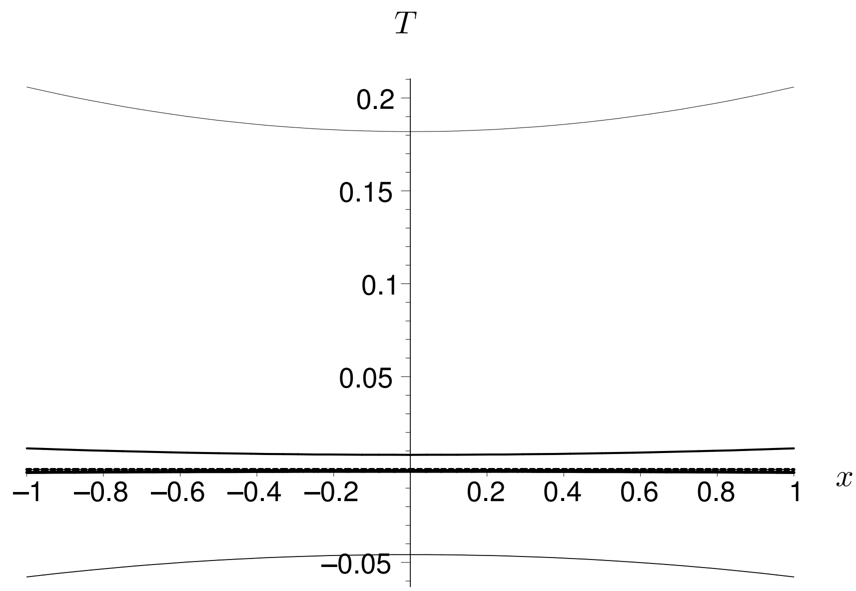
$$T_{\mathrm{LS}}(N) = 1 - \sum_{n=0}^{N} (-1)^n \left[\operatorname{erfc}\left(\frac{2n+1-x}{2\sqrt{t}} \right) + \operatorname{erfc}\left(\frac{2n+1+x}{2\sqrt{t}} \right) \right],$$

$$T_{\mathrm{FS}}(N) = \sum_{n=0}^{N} \frac{4(-1)^n}{(2n+1)\pi} \exp\left(-\left(n + \frac{1}{2} \right)^2 \pi^2 t \right) \cos\left(\left(n + \frac{1}{2} \right) \pi x \right).$$

The claim was that the "Laplace series" (LS) was better for small times, while the Fourier series (FS) was better for large times. The figure below shows that the Laplace series is superior for small times. The figure on the reverse shows that the Fourier series is superior for large times.



Temperature *vs.* x for $t = 10^{-3}$. Dotted line: $T_{\mathrm{LS}}(1)$.
In increasing order of thickness: $T_{\mathrm{FS}}(1)$, $T_{\mathrm{FS}}(3)$, $T_{\mathrm{FS}}(5)$, $T_{\mathrm{FS}}(7)$.



Temperature *vs.* x for $t = 5$. Dotted line: $T_{\mathrm{FS}}(1)$.
In increasing order of thickness: $T_{\mathrm{LS}}(1)$, $T_{\mathrm{LS}}(2)$, $T_{\mathrm{LS}}(3)$, $T_{\mathrm{LS}}(4)$.