Complementary Slackness Conditions

Consider the following primal-dual pair:

\[
\begin{pmatrix}
3 & -1 & 1 \\
1 & 2 & 5 \\
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
\end{pmatrix} =
\begin{pmatrix}
2 \\
3 \\
\end{pmatrix}, \quad x \geq 0, \quad \max U(x) = (7, 0, 3)^T x. \quad (P)
\]

\[
\begin{pmatrix}
3 & 1 \\
-1 & 2 \\
1 & 5 \\
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2 \\
\end{pmatrix} \geq 
\begin{pmatrix}
7 \\
0 \\
3 \\
\end{pmatrix}, \quad \min V(y) = 2y_1 + 3y_2. \quad (D)
\]

The graph of the primal solution is shown above; \( U \) increases as we move out of the plane of the paper. The basic solutions are the endpoints: \((1/2, 0, 1/2)\), where \( U = 5 \), and \((1, 1, 0)\), where \( U = 7 \).
The graph of the dual solution is shown above. The feasible region is shaded; the
dotted lines show level curves of $V$ (thicker dots mean larger values). Hence the minimum
is at $(2, 1)$, as derived. The dashed line is the third constraint $y_1 + 5y_2 = 3$, which is never
active (hence $x_3 = 0$ for the optimal primal solution).