Updates

1. Honors topics are due Monday, Sept. 28.

Homework Set 4 (Revised)


Utility Functions

1. page 172, exercise 7-P
   (a) (5 points) part 1. As an intermediate step, you should show that

   \[
   \frac{\partial x^*_j}{\partial p_j} = -\frac{x^*_j}{p_j}, \quad \frac{\partial x^*_i}{\partial p_i} = -\frac{x^*_i}{p_j},
   \]

   where the $x^*_j$ and $p_j$ are components of the optimal bundle and price vector, respectively.

   (b) (3 points) part 2.

2. (6 points per part) For the following relations on the listed spaces, determine the conditions (if any) on the listed characteristic that:
   - make the relation a preference relation,
   - make the preference relation complete,
   - make the complete preference relation strongly monotone.

   In the case of a complete strongly monotone preference relation, write a corresponding utility function. (You may assume that any preference relation which satisfies all the other properties is continuous.)

   (a) characteristic: set $X$

   \[ X \subseteq \mathcal{R}^n, \quad x \succeq y \text{ if and only if } \min_i x_i \geq \max_i y_i. \]

   (b) characteristic: vector $z$

   \[ X = \{ x \in \mathcal{R}^n | x \geq 0 \}, \quad x \succeq y \text{ if and only if the distance between } x \text{ and } z \text{ is greater than or equal to the distance between } y \text{ and } z, \text{ for some fixed } z \in \mathcal{R}^n. \]

   (c) characteristic: set $X$

   \[ X \subseteq \mathcal{R}^n, n \text{ odd}; \quad x \succeq y \text{ if and only if } \prod_{i=1}^{n} x_i \geq \prod_{i=1}^{n} y_i. \]
Concave Functions

3. Let \( f(x) \in C^\infty[a, b] \) \( i.e. \), \( f(x) \) has infinitely many continuous derivatives in \([a, b]\). Moreover, let \( f(x) \) be strictly concave.

(a) (6 points) Show that wherever \( f''(x) \neq 0 \) for some \( x \in [a, b] \), \( f''(x) < 0 \).
(b) (3 points) Show that \( f''(x) = 0 \) only at isolated points in \([a, b]\).

4. (5 points) Prove Theorem CF1 in the other direction. In particular, let \( X \subseteq \mathbb{R}^n \), \( f : X \rightarrow \mathbb{R} \). Moreover, assume that for any \( x_i \in X \),

\[
\sum_{i=1}^{n} \lambda_i f(x_i) \leq f \left( \sum_{i=1}^{n} \lambda_i x_i \right), \quad \lambda_i \in [0, 1], \quad \sum_{i=1}^{n} \lambda_i = 1. \tag{4.2}
\]

Prove that \( \text{hyp} \ f \) is convex.