Homework Set 3 (Revised)

Read the intro to chapter 5, sections 7.1–7.3, and section A-5.

Applications

1. Suppose that a household wants to purchase quantities $x_i$ of two items, where the price per item follows the following law:

$$p(x_i) = \max\{20 - x_i, 14\}.$$

(a) (5 points) What is the maximum number of items $x_i$ you can buy before the cost function becomes nondifferentiable? Explain how this implies that for some budgets, the boundary of the opportunity set will be smooth (differentiable), while for others, it will not. Find the value of the budget constraint that divides the two cases.

(b) (3 points) Graph the opportunity set $X$ for each case.

(c) (7 points) Determine whether $X$ is convex.

2. Suppose that in addition to the monetary budget constraint

$$p^T x \leq B, \quad p \geq 0,$$

there is also a “point budget” constraint

$$q^T x \leq C, \quad q \geq 0.$$  \hspace{2cm} (3.1a)

These “points” could be ration coupons (as used during wartime), Weight Watchers points, calorie counts, and so on.

(a) (3 points) Prove that the opportunity set subject to both constraints (3.1) is convex.

(b) (7 points) Let $p = (10, 1)$, $B = C = 10$. Draw the opportunity set for various $q$. You should find four cases, each of which holds for infinitely many $q$. Explain each economically. If points are given out equally to all, what system would allow for the widest distribution of scarce goods?

Preference Relations

3. page 170, exercise 7-C (a: 2 points; b: 5 points)

4. (4 points) Show that the indifference relation is an equivalence relation.
5. (4 points) Let $\succeq$ be a complete, strongly monotonic preference relation. Moreover, let $x, y \in X$, the opportunity set. Show that if $x \succ y$ for all $y \neq x$, $x$ must lie on the budget line.