Updates

1. Exam II will be administered Wednesday, Nov. 18. The exam will cover up through pure strategies so you should do the first question before the exam for practice.
2. You will be allowed one letter-size page of notes to use on the test.
3. Honors papers are due Monday, Nov. 30.

Homework Set 10

Pure Strategies

1. Consider the two-person, zero-sum game diagrammed above, which depends on the parameter $\alpha$.
   
   (a) (7 points) There are six open intervals of $\alpha$ to consider. For each:
   (i) Determine whether the game has a solution in pure strategies,
   (ii) Identify those pure strategies, and
   (iii) Determine the value of the game.
   
   (b) (5 points) Now consider the five values of $\alpha$ which bound the six intervals of part (a). Use your answers to show that there are four intervals of $\alpha$ which determine different outcomes of the game.
Mixed Strategies

2. Consider the following zero-sum game: players 1 and 2 independently choose a number from \{1, 2, 3\}. If the numbers are equal, player 1 pays player 2 that amount. If the numbers are unequal, player 2 pays player 1 the amount corresponding to the number player 2 chose.

(a) (3 points) Write the matrix form for this game.
(b) (7 points) Solve the game. If you use technology to solve the linear system, please include a printout. (Hint: \( v = 6/11 \).)

Nonzero-Sum Games

3. Consider a game between two players, as shown above. However, in this case the players are partners and each is willing to sacrifice his/her own payoff if it will increase the payoff of the other player. Suppose that (as in the Prisoner’s Dilemma), each player initially starts with strategy 1.

(a) (3 points) For what values of the parameters will the players switch strategies?
(b) (3 points) For what values of the parameters will the game result in the Altruist’s Dilemma, in which both players are worse off than when they started?

Jensen’s Inequality

4. Consider the function
\[
p(w) = \frac{\alpha}{w} \exp \left( \beta \log^2 w \right), \quad w > 0.
\]

(a) (9 points) Find conditions under which \( p(w) \) is a probability density function, and calculate its mean.
(b) (3 points) Verify Jensen’s Inequality explicitly with \( u(w) = \log w \) and \( p(w) \) as defined using your answer to (a).