Phase Plane: Real Eigenvalues

For the system

\[ \dot{x} = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} x, \quad (1) \]

the solution is

\[ x = c_1 e^{-5t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \]

Since we have one positive and one negative eigenvalue, we have a saddle point, as shown below. Note the straight lines corresponding to the eigenvectors.

Phase plane of (1).
For the system
\[
\dot{x} = \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix} x,
\]
(2)
the solution is
\[
x = c_1 e^{-4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
\]
Since we have two negative eigenvalues, we have a stable node, as shown below.

Phase plane of (2).