Homework Set 5

Read sections P2.2, P3.1, P3.2.

Section P2.2

1. Solve for the currents in the network above.

2. Consider the system

\[ 3x + y - 2z = 2, \]
\[ -x + 4y + z = -1, \]
\[ 7x + 11y - 4z = 4. \]

(a) Use Gaussian elimination to reduce the system to row echelon form.
(b) Is the system overdetermined? underdetermined? inconsistent?
(c) Transform the system to reduced row echelon form and write the solution.

3. Let \( A \in \mathbb{R}^{m \times n}, \ b \in \mathbb{R}^{m}, \ x \in \mathbb{R}^{n}. \) Moreover, let \( Ax = b, \) and \( z = -5b. \) Does the equation \( Ay = z \) have a solution? If so, compute it. If not, explain why not.
4. Consider the system
\[
\begin{align*}
x - y + z &= 3 \\
-2x + 3y + z &= -8 \\
4x - 2y + 10z &= b,
\end{align*}
\]
where \(b\) is a constant.
(a) Write the system in augmented matrix form, and row reduce it to the following form:
\[
\begin{pmatrix}
1 & 0 & * & * \\
0 & 1 & * & *
\end{pmatrix},
\]
where the * are unknown entries that you must find.
(b) Find all values of \(b\) for which (5.1) has a solution, and write the solution in that case.

Sections P3.1/3.2

5. (BH) Consider the following matrices:
\[
A = \begin{pmatrix} 1 & 2 & -4 \\ -2 & -3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 3 & -2 \\ -1 & -4 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -2 & 0 \\ 0 & -2 & 3 \\ 1 & -2 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} -2 & 1 \\ 2 & -2 \end{pmatrix}, \quad E = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 2 & -2 \\ 1 & 3 & 1 \end{pmatrix}.
\]
If possible, compute
(a) \(A(BD)\)
(b) \((AB)D\)
(c) \(A(C + E)\)
(d) \(AC + AE\)

6. Let \(A \in \mathbb{R}^{m \times n}\).
(a) Verify that \(AA^T\) exists and calculate its size.
(b) Show that if \(A \in \mathbb{R}^{m \times n}\), \(AA^T\) must be symmetric.

7. Let \(A\) and \(B\) be symmetric matrices. Prove that \(AB = BA\) if and only if \(AB\) is also symmetric.
8. Consider the following matrices:

\[
A = \begin{pmatrix}
2 & 1 \\
6 & 3 \\
-2 & 4
\end{pmatrix}, \quad B = \begin{pmatrix}
2 & 4 \\
1 & 6
\end{pmatrix}.
\]

(a) Verify that \(3(AB) = (3A)B = A(3B)\) and \((AB)^T = B^TA^T\).
(b) For the \(A\) and \(B\) defined in this problem, calculate the following (or indicate it doesn’t exist):
\[A^TB, \quad B^TA, \quad AB^T.\]

9. Consider the linear system
\[
\begin{align*}
x_1 + x_2 + x_3 &= 5, \\
-2x_1 - 3x_2 + 2x_3 &= 4, \\
4x_1 + 2x_2 + x_3 &= 4.
\end{align*}
\]

(a) Write the system (5.2) as a matrix-vector equation.
(b) Solve the system (5.2).

10. Let
\[
A = \begin{pmatrix}
3 & -1 \\
2 & 0
\end{pmatrix}, \quad b = \begin{pmatrix}
3 \\
4
\end{pmatrix}, \quad c = \begin{pmatrix}
7 \\
-4
\end{pmatrix}.
\]

(a) Write \(b\) as a linear combination of the columns of \(A\).
(b) Use your solution to (a) to solve the system \(Ax = b\).
(c) Write \(c\) as a linear combination of the columns of \(A\).