MATH 349-011 Elementary Linear Algebra
Prof. D. A. Edwards Due: Nov. 12, 2009

Homework Set 8

Read sections 3.6, 6.4, 6.5.

Sections 3.6/6.4

1. (BH) page 484, exercise 20
2. (BH) page 485, exercise 22
3. (BH) page 485, exercise 28
4. (BH) Let $\mathcal{L} : \mathcal{R}^2 \to \mathcal{R}^2$ be a linear transformation for which we know that

$$\mathcal{L}\left\{\begin{pmatrix}2\\1\end{pmatrix}\right\} = \begin{pmatrix}3\\-2\end{pmatrix}, \qquad \mathcal{L}\left\{\begin{pmatrix}-1\\-2\end{pmatrix}\right\} = \begin{pmatrix}1\\1\end{pmatrix}.$$

   (a) What is $\mathcal{L}\{(4,-1)^T\}$?
   (b) What is $\mathcal{L}\{(a_1,a_2)^T\}$?
5. (BH) Decide whether the following transformations are linear. Indicate your reasoning.
   (a) $T : \mathcal{R}^3 \to \mathcal{R}^2$, where $T(\mathbf{x}) = (3x_3 - 2x_2, -x_1)^T$.
   (b) $T : \mathcal{R}^2 \to \mathcal{R}^2$, where $T(\mathbf{x}) = (|x_1 + x_2|, 3x_1 - x_2)^T$.
   (c) $T : \mathcal{R}^2 \to \mathcal{R}^2$, where $T(\mathbf{x}) = (4x_1 + x_2, 0)^T$.
   (d) $T : \mathcal{R}^2 \to \mathcal{R}$, where $T(\mathbf{x}) = 3x_1 - 2x_2 + 1$.

Section 6.5

6. (BH) page 500, exercise 20
7. (BH) Let $\mathcal{L} : \mathcal{P}_2 \to \mathcal{P}_1$ be the linear transformation defined by

$$\mathcal{L}(at^2 + bt + c) = (3a - b)t + (a + c).$$

   (a) Find a basis for $\ker \mathcal{L}$.
   (b) Find a basis for $\mathcal{R}(\mathcal{L})$.

8. (BH) Let $C_1(I)$ be the space of all functions that are continuously differentiable functions on some interval I containing $x = 1$, and let $\psi(x) \in C_1(I)$. Consider the following two linear transformations, which map $C_1(I)$ into itself:

$$D(\psi) = \frac{d\psi}{dx}, \qquad T(\psi) = x\psi.$$

   (a) Show that $D(T(\psi)) - T(D(\psi)) = \psi$. (In other words, $(D \circ T) - (T \circ D)$ is the identity transformation.)
   (b) Describe the kernel and range of $T \circ D$.

   Now let $A, B \in \mathcal{R}^{n\times n}$.

   (c) Show that $AB - BA \neq I$ for any matrices A and B. (*Hint: Consider the trace, which is the sum of the diagonal entries.*)

9. (BH) For each of the following linear transformations, determine its kernel and range:
   (a) $L : \mathcal{R}^3 \to \mathcal{R}^3$ projects every point $\mathbf{x}$ to the plane $x_1 = x_2$. (This is an orthogonal projection.)
   (b) $L : \mathcal{R}^2 \to \mathcal{R}^2$ reflects a vector about the x_1-axis and then rotates it by $-\pi/4$.
10. (BH) Determine the kernel and range of each of the transformations in #5.