MATH 349-011 Elementary Linear Algebra
Prof. D. A. Edwards Due: Nov. 5, 2009

Updates

1. Exam II will be administered Tuesday, Oct. 27. You will need to bring a small blue book.
2. Due to the intervening exam, this homework set is not due until Thursday, Nov. 5. However, to prepare for the exam you should do the first four problems before the exam date.
3. The final for this class will be administered on Thursday, Dec. 17 from 1–3 in the normal classroom.

Homework Set 7

Read sections 5.1–5.3, 7.1, 7.3.

Section 5.1

1. (BH) Consider the set

$$\left\{ \begin{pmatrix} 3 \\ -2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 8 \\ 7 \\ 0 \end{pmatrix} \right\}.$$

   (a) Verify that the set is orthogonal.
   (b) Normalize the vectors to create an orthonormal set.

2. Let

$$\langle f, g \rangle = \int_0^{\pi} f(x) g(x)\, dx.$$

   (a) (BH) Let $f(x) = x^{1/3}$. Find *all* quadratic functions

$$g(x) = \frac{4a}{3} + \frac{7bx}{3} + \frac{10cx^2}{3}$$

   such that f and g are orthogonal.
   (b) (MP) Let $f(x) = \cos x$. Find *all* values α for which $f(x)$ and $\cos \alpha x$ are orthogonal.

Section 5.2

3. (BH) Let

$$W = \text{Span}\left\{ \begin{pmatrix} 1 \\ -3 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} -2 \\ 6 \\ 0 \\ -3 \end{pmatrix} \right\}.$$

   Find a basis for $W^\perp$.

4. (BH) Consider the matrix

$$A = \begin{pmatrix} 1 & -2 & 3 & 5 \\ 1 & -1 & 8 & 7 \\ 2 & -4 & 6 & 10 \end{pmatrix}.$$

   Find the four fundamental subspaces associated with A and verify Theorem 5.10 on p. 376.

Section 5.3

5. (BH) Use the Gram-Schmidt procedure to find an *orthonormal* basis for the following subspace of $\mathcal{R}^4$:

$$W = \left\{ \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix}, w_1 + 2w_2 + 5w_3 - 60w_4 = 0 \right\}.$$

6. Consider the following matrices:

$$B = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 0 & 4 \\ 1 & 1 & 5 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & 1 & -3 & 2 \\ 3 & 2 & -1 & 7 \\ 1 & -1 & -2 & -1 \\ -4 & 2 & -1 & -7 \\ 7 & -3 & -3 & 8 \end{pmatrix}.$$

   (a) (BH) Show that $\operatorname{col} B = \mathcal{R}^3$.
   (b) (BH) Using the Gram-Schmidt procedure on the columns of B, find an *orthonormal* basis for the column space of B.
   (c) (MP) Using the Gram-Schmidt procedure, find an *orthonormal* basis for the column space of C.

Section 7.1

7. (BH) Define an inner product on $\mathcal{P}_2$ by

$$\langle p(t), r(t)\rangle = p(-1)r(-1) + p(0)r(0) + p(1)r(1).$$

Let $p_0(t) = t^2$, $p_1(t) = t$, and $p_2(t) = 1$.

(a) Compute the orthogonal projection of p_2 onto the subspace spanned by p_0 and p_1.

(b) Find a polynomial $s(t)$ that is orthogonal to p_0 and p_1 such that $\{p_0, p_1, s\}$ is an orthogonal basis for $\text{Span}\{p_0, p_1, p_2\}$.

8. (BH) Apply the Gram-Schmidt procedure to the basis $\{3, -6t, 6t^2\}$ for the vector space $\mathcal{P}_2$ and obtain an orthonormal basis for $\mathcal{P}_2$ using

$$\langle f, g\rangle = \int_{-1}^{1} fg\,dt.$$

Section 7.3

9. (BH) Suppose that a certain linear algebra professor has been strapped to a medieval rack.
Under a force of 1 ton, his length is stretched to 190 cm.
Under a force of 3 tons, his length is stretched to 220 cm.
Under a force of 5 tons, his length is stretched to 250 cm.
If we consider his stretched corpse to be a spring, then according to Hooke's Law, his length L under a force F would be given by

$$L = a + bF,$$

where a and b are constants.

(a) Given the data above, find a and b.

(b) Using Hooke's Law, what was the professor's original height?

10. (MP) Consider the following data for the U. S. debt:

Year	Total Debt (in billions)	Year	Total Debt (in billions)
1996	$5,225	2001	$5,807
1997	$5,413	2002	$6,228
1998	$5,526	2003	$6,783
1999	$5,656	2004	$7,379
2000	$5,674	2005	$7,933

(a) Find the line that best fits the data.

(b) Project the debt in 2008, 2013, and 2020.

(c) Find a debt calculation for 2008 on the Internet and compare it to your calculation in (b). What can you conclude?