MATH 349-011 Elementary Linear Algebra
Prof. D. A. Edwards Due: Oct. 22, 2009

Homework Set 6 (Revised)

Read sections 6.3, 7.1, 7.2.

Section 6.3

1. (BH) Compute the coordinates of $\mathbf{v} = 7t^2 - 29$ in the ordered basis $S = \{t^2 + 3t - 1, 2t + 4, t^2 - 3\}$.
2. (MP) Consider the basis

$$B = \left\{ \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \\ 5 \end{pmatrix}, \begin{pmatrix} 9 \\ 1 \\ 8 \end{pmatrix} \right\}.$$

   Find the transition matrix from the standard basis to B-coordinates.
3. (BH) Let

$$S = \left\{ \begin{pmatrix} -1 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \end{pmatrix} \right\}, \qquad T = \left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\},$$
$$\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}.$$

   (a) Find the coordinate vectors of $\mathbf{v}$ and $\mathbf{w}$ with respect to the basis T.
   (b) What is the transition matrix ${}_{S\leftarrow}P_{T}$ from the T- to the S-basis?
   (c) Find $[\mathbf{v}]_S$ and $[\mathbf{w}]_S$ using ${}_{S\leftarrow}P_{T}$.
   (d) Find $[\mathbf{v}]_S$ and $[\mathbf{w}]_S$ directly.
   (e) Find the transition matrix ${}_{T\leftarrow}P_{S}$ from the S- to the T-basis.
   (f) Find $[\mathbf{v}]_T$ and $[\mathbf{w}]_T$ using ${}_{T\leftarrow}P_{S}$. Compare the answers with those in (a).

Section 7.1

4. (BH) page 550, exercise 34
5. (BH) page 551, exercise 44

6. (BH) Consider each of the following pairs as vectors in the vector space $\mathcal{P}_2$. Calculate $\langle f, g\rangle$ for each, where

$$\langle f, g\rangle = f(-1)g(-1) + f(0)g(0) + f(1)g(1).$$

   (a) $f(t) = 3 + t$, $g(t) = 5 - t$
   (b) $f(t) = g(t) = t^2 - 2t + 1$
   (c) $f(t) = t^2 - 1$, $g(t) = t + 1$

7. (BH) Consider each of the following pairs as vectors in the vector space $C[0, 1]$. Calculate $\langle f, g\rangle$ for each, where

$$\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt.$$

   (a) $f(t) = 3 + t, \qquad g(t) = 5 - t$
   (b) $f(t) = \sin \pi t, \qquad g(t) = \cos \pi t$
   (c) $f(t) = t, \qquad g(t) = \dfrac{1}{t^2 + 1}$

Section 7.2

8. (BH) page 576, number 16
9. Let $\mathbf{v} = (2, 1, -2, 0, 4)^T$.
   (a) (BH) Calculate $||\mathbf{v}||_1$, $||\mathbf{v}||_2$, and $||\mathbf{v}||_3$.
   (b) (MP) Calculate $||\mathbf{v}||_{10}$, $||\mathbf{v}||_{20}$, and $||\mathbf{v}||_{30}$. Express your answers as decimals.
   (c) (BH) Your answer to (b) should you lead to suspect that

$$\lim_{p\to\infty} ||\mathbf{v}||_p = ||\mathbf{v}||_\infty$$

   for any $\mathbf{v}$. (This norm is also denoted $||\mathbf{v}||_m$ in your book.) Prove it.

10. (BH) Determine whether the following functions are norms for $\mathcal{C}[-1, 1]$:

   (a) $||f|| = 3(|f(-1)| + |f(1)|)$
   (b) $||f|| = \displaystyle\int_{-1}^{1} f(x)\,dx$
   (c) $||f|| = \left[\displaystyle\int_{-1}^{1} f^2(x)\,dx\right]^{1/2}$

(*Hint: To check the Triangle Inequality in (c), relate the norm to an inner product.*)