MATH 349-011 Elementary Linear Algebra
Prof. D. A. Edwards Due: Oct. 15, 2009

Homework Set 5

Read sections 3.5, 6.2.

Section 3.5

1. (BH) page 209, exercise 60
2. (BH) Let $A \in \mathcal{R}^{2\times 6}$.
   (a) Give *all* possible values for $\operatorname{rank} A$.
   (b) If $\operatorname{rank} A = 2$, what is the dimension of its column space?
   (c) If $\operatorname{rank} A = 2$, what is the dimension of the solution space of the homogeneous system $A\mathbf{x} = \mathbf{0}$?
3. (BH) Let
$$A = \begin{pmatrix} -1 & 2 & -4 & 3 & 5 \\ 4 & 5 & 3 & 14 & -7 \\ 2 & -3 & 7 & -4 & -9 \end{pmatrix}.$$
   (a) Write a basis for $\mathcal{N}(A)$, the null space of A.
   (b) Write a basis for $\operatorname{col} A$.
   (c) Write a basis for $\operatorname{row} A$. Verify that $\dim \operatorname{row} A = \dim \operatorname{col} A$.
   (d) Verify that $\dim \mathcal{N}(A) + \operatorname{rank} A = n$, the number of columns of the matrix.
4. (MP) Repeat the steps in #3 for the matrix
$$A = \begin{pmatrix} 2 & 1 & 3 & 4 & 1 \\ 0 & -1 & 1 & 2 & -1 \\ -1 & -4 & 2 & 1 & 0 \\ 3 & 7 & -1 & 1 & 1 \end{pmatrix}.$$
5. (BH) Let
$$B = \begin{pmatrix} -7 & -4 & 1 \\ -4 & 2 & -8 \\ -5 & 0 & -5 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$
   (a) Is $\mathbf{v} \in \operatorname{col} B$?
   (b) Show that $\mathbf{v} \in \mathcal{N}(B)$.
   (c) Explain why there must be a vector $\mathbf{w} \in \mathcal{R}^3$, $\mathbf{w} \neq \alpha\mathbf{v}$, such that $B^2\mathbf{w} = \mathbf{0}$.

6. (BH) Consider the following two sets of equations:

$$\begin{aligned} 232x - 101y + 75z &= 2 \\ 311x + 137y + 264z &= 4 \\ 141x - 632y + 511z &= -5 \end{aligned} \qquad \begin{aligned} 232x - 101y + 75z &= -6 \\ 311x + 137y + 264z &= -12 \\ 141x - 632y + 511z &= 15 \end{aligned}$$

The system on the left has a solution (you can take this as a given). Explain why the system on the right *must* have a solution. Do **NOT** calculate either solution; just explain why it must be so given the theory you have learned in class.

Section 6.2

7. (BH) page 461, exercise 20. If the answer is no, write a basis for Span $\mathcal{B}$.
8. (BH)
   (a) page 461, exercise 22
   (b) page 461, exercise 24
9. (BH)
   (a) page 462, exercise 40
   (b) Find a basis for the vector space of all $n \times n$ symmetric matrices.
10. (BH) Determine whether each of the following sets is linearly independent in the appropriate vector space:

(a) $$\left\{ \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 8 \\ -6 \end{pmatrix} \right\}$$

(b) $$\left\{ \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \right\}$$

(c) $\{\sin^2 x, \cos 2x, 1\}$

(d) $\{3, 7 - x^2, 4 + x, 2 - x^3\}$