MATH 349-011 | Elementary Linear Algebra
Prof. D. A. Edwards | Due: Oct. 8, 2009

Updates

1. Exam I will be administered Thursday, Oct. 1. You will need to bring a small blue book. The exam will cover up through today's lecture (sections 3.1 and 3.2).
2. Due to the intervening exam, this homework set is not due until Thursday, Oct. 8. However, to prepare for the exam you should do #1–5 before the exam date.

Homework Set 4 (Revised)

Read sections 3.1–3.3, 3.5, 6.1.

Sections 3.1/3.2

1. (BH) page 151, exercise 30
2. (BH) page 160, exercise 36
3. (BH) Consider the following matrices:

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -4 & 1 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -1 \\ 2 & -2 \\ -3 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 2 & 0 \\ -3 & -2 & -2 \\ 1 & 1 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix}, \qquad E = \begin{pmatrix} 1 & 2 & -4 \\ 5 & 0 & -5 \\ -1 & 3 & 3 \end{pmatrix}, \qquad F = \begin{pmatrix} 0 & -1 \\ 1 & -3 \\ 3 & 2 \end{pmatrix}.$$

If possible, compute

(a) $A(BD)$
(b) $(AB)D$
(c) $A(C+E)$
(d) $AC+AE$
(e) $3A+2A$ and $5A$
(f) $A(C-3E)$

4. (MP) Consider the following matrices:

$$A = \begin{pmatrix} 2 & 4 \\ -5 & 1 \\ 1 & -3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}.$$

Verify that

(a) $-2(B + C) = -2B - 2C$

(b) $A(B - C) = AB - AC$

5. (BH) Let

$$A = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}, \qquad C = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}.$$

(a) Compute $(AB^T)C$.

(b) Compute $B^T C$ and multiply the result by A on the right.

(c) Explain why $(AB^T)C = (B^T C)A$.

Section 3.3

6.

(a) (MP) Construct the inverse of

$$A = \begin{pmatrix} 3 & 2 & -1 & 4 \\ 2 & 5 & -3 & -1 \\ -5 & 0 & 4 & 3 \\ 2 & 1 & -4 & 7 \end{pmatrix}.$$

(b) (MP) Consider the matrix

$$B = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 3 & -4 \\ 5 & -2 & 2 \end{pmatrix}.$$

Show that $(B^{-1})^T = (B^T)^{-1}$.

(c) (BH) page 177, exercise 52

(d) (BH) Construct the inverse of

$$D = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 4 & 3 & 1 \end{pmatrix}.$$

7. (BH) For each pair of matrices $\{A, B\}$, find an elementary matrix E such that $EA = B$. Also, explain in words what row operations each elementary matrix performs.

$$A_1 = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}, \tag{a}$$

$$A_2 = \begin{pmatrix} 2 & -3 \\ 4 & 2 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 1 & -3/2 \\ 4 & 2 \end{pmatrix}, \tag{b}$$

$$A_3 = \begin{pmatrix} 2 & -5 & 1 \\ 1 & 3 & 3 \\ 2 & -1 & 4 \end{pmatrix}, \qquad B_3 = \begin{pmatrix} 2 & -5 & 1 \\ 0 & -7 & -2 \\ 2 & -1 & 4 \end{pmatrix}. \tag{c}$$

Section 3.5/6.1

8. (BH) In each part, consider the given subset of the vector space P_2. Is the subset a subspace?
   (a) page 446, exercise 34
   (b) page 446, exercise 36
   (c) $\{a_2x^2 + a_1x + a_0, \quad a_2 = -3a_1\}$
9. (BH) Let V be the set of all ordered pairs of real numbers (x_1, x_2). Define vector addition in the normal way, but define scalar multiplication by $c \odot (x_1, x_2) = (cx_1, x_2)$. Check to see if this combination is a vector space. Check all properties, noting which hold and which do not.
10. Consider the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ -1 & -2 & 2 & -2 \\ 2 & 3 & 0 & 3 \end{pmatrix}.$$

   (a) (BH) Find a basis for $\mathcal{N}(A)$.
   (b) (BH) Show that $\mathcal{N}(A)$ satisfies the definition of a subspace, and find m, where $\mathcal{N}(A)$ is a subspace of $\mathcal{R}^m$.
   (c) (MP) Find the null space of

$$\begin{pmatrix} 7 & 2 & 1 & 5 & 20 \\ -12 & -13 & -5 & -2 & 15 \\ 11 & 0 & 6 & 18 & 22 \\ 5 & -4 & 3 & 5 & -2 \end{pmatrix}.$$