Homework Set 3 (Revised)

Read sections 2.2, 2.4.

Section 2.2

1. (BH) Solve the linear system

$$\begin{aligned} x + 2y - z &= 3 \\ 3x + 6y + 2z &= -1 \\ -4x - 8y - 6z &= 8. \end{aligned}$$

2. (BH) Solve the linear system

$$\begin{aligned} 3x + 6y - 3z &= 6 \\ -2x - 3y + 2z &= 1 \\ 4x + 9y - 4z &= 0. \end{aligned}$$

3. (BH) Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 3 & 1 & 1 & 2 \\ -2 & -2 & 4 & -1 \\ -1 & 5 & 2 & 3 \end{pmatrix}.$$

(a) Find matrices B and C in row echelon form that are row equivalent to A.
(b) Find a matrix D in reduced row echelon form that is row equivalent to A.

4. (MP) Solve the linear system with the given augmented matrix, or show it has no solution.

$$\text{(a)}: \quad \left(\begin{array}{rrrr|r} 1 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & 1 & 4 \\ -1 & -2 & 2 & 4 & 5 \end{array}\right)$$

$$\text{(b)}: \quad \left(\begin{array}{rrrr|r} 2 & 5 & 5 & 3 & 4 \\ 1 & 4 & 3 & -2 & 3 \\ 4 & 7 & 9 & 13 & 0 \end{array}\right)$$

5. (BH) Consider the following system:

$$x_1 + ax_2 = 4, \tag{3.1a}$$
$$2x_1 + 2x_2 = b. \tag{3.1b}$$

   (a) Write (3.1) as an augmented matrix.
   (b) Write your answer to (a) in *reduced* row echelon form.
   (c) For which value(s) of a and b will (3.1) have
      (i) more than one solution?
      (ii) exactly one solution?
      (iii) no solution?

Section 2.4

6. (BH) page 114, number 10
7. page 114, number 18. Do part (a) with Maple and the rest by hand.
8. (BH) page 118, number 40
9. (BH) You are given three alloys of the following composition:
   Alloy A: 4 parts (by weight) gold, 1 silver, 2 zinc
   Alloy B: 2 parts gold, 3 silver, 2 zinc
   Alloy C: 2 parts gold, 1 silver, 4 zinc
   (a) How much of each metal is in one ounce of Alloy A?

   Suppose we want to make 18 ounces of a new alloy containing equal quantities (by weight) of gold, silver, and zinc.

   (b) Write the system of equations needed to solve for the amounts of A, B, and C necessary to make this new alloy.
   (c) Solve the system.
10. Suppose that the following facts are true:
   (1) Of the number of people x_{s} who start a year living in the Midwest, 90% stay there and 10% move out during the course of that year.
   (2) Of the number of people y_{s} who start a year living outside of the Midwest, 95% stay out and 5% move in during the course of that year.
   (3) The number of births and deaths cancel one another out (so you don't have to worry about them).

   Let x_{e} and y_{e} be the number of people living in and out of the Midwest, respectively, at the *end* of the year.

   (a) (BH) Write down equations needed to solve for x_{e} and y_{e} as functions of x_{s} and y_{s}.

   The system in (a) is an example of a *Markov process.*

   (b) (MP) If $y_{\text{e}} = 232$ million and $x_{\text{e}} = 66$ million, find x_{s} and y_{s}.
   (c) (BH) If $x_{\text{s}} = x_{\text{e}}$ and $y_{\text{s}} = y_{\text{e}}$, find the ratio $y_{\text{e}}/x_{\text{e}}$.

   In the case given by (c), $(x_{\text{s}}, y_{\text{s}})$ is called an *eigenvector* of the linear system. We will study this concept in great detail later.