MATH 349-011 Elementary Linear Algebra
Prof. D. A. Edwards Due: Sept. 10, 2009

Use of Maple

Each problem has a code that indicates how you are to use Maple on your submitted work. (You may always use Maple to check your work.) The codes are:

BH: (by hand) anything submitted should be done by hand and should show all steps.

MP: (Maple printout) use Maple as appropriate to get the answer and give me a printout of your worksheet.

Homework Set 1 (Revised)

As I mentioned in class, the material covered this week will be scattered throughout the book. Here are the pages you should read. If you come to something that wasn't covered in class this week, skip it; we will come back to it later on in the course.

Read §1.1; §2.3; pp. 196, 200, 201 (§3.5)

Section 1.1

1. Let
$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}, \qquad \mathbf{d} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$
   (a) (MP) Compute the coordinates of $\mathbf{a} + \mathbf{b}$ and $\mathbf{c} - \mathbf{d}$.
   (b) (BH) Sketch the vectors $\{\mathbf{a}, \mathbf{b}, \mathbf{a} + \mathbf{b}\}$ and $\{\mathbf{c}, \mathbf{d}, \mathbf{c} - \mathbf{d}\}$ and compute the coordinates geometrically.
2. (BH) page 14, exercise 18
3. (BH) Let
$$\mathbf{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} 4 \\ r \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} s \\ 2 \end{pmatrix}.$$
   Find r and s so that
   (a) $\mathbf{z} = -2\mathbf{x}$.
   (b) $3\mathbf{u}/2 = \mathbf{y}$.
   (c) $\mathbf{z} - \mathbf{u} = \mathbf{x}$.

4. (BH) A box of cereal contains a mixture of b grams of bran flakes and c grams of corn flakes. The specifications of each are as follows:

Property	bran flakes (per gram)	corn flakes (per gram)
calories	3.2	3.6
fiber (g)	0.17	0.04
Iron (% of daily allowance)	1.5	1.1

   (a) How many calories are in the box?
   (b) We want to express the content of the box as a three-dimensional vector consisting of calories, fiber, and iron. Express this output as a linear combination of two vectors.

Section 2.3

5. (BH) page 101, number 46
6. (BH) Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors in a vector space V. Prove that $T = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ is also linearly independent, where $\mathbf{w}_1 = \mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3$, $\mathbf{w}_2 = \mathbf{v}_2 + 2\mathbf{v}_3$, and $\mathbf{w}_3 = 4\mathbf{v}_3$.
7. (BH) Show that

$$\text{Span}\left\{\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}\right\} = \text{Span}\left\{\begin{pmatrix} 2 \\ -5 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 12 \\ -5 \end{pmatrix}\right\}.$$

8. (BH) Consider the following vectors:

$$\mathbf{x}_1 = \begin{pmatrix} -3 \\ 1 \\ 6 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}, \quad \mathbf{x}_3 = \begin{pmatrix} 1 \\ z \\ -2 \end{pmatrix}.$$

   (a) For which value(s) of z does the equation $\mathbf{x}_3 = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$ have a solution with either $c_1 \neq 0$ or $c_2 \neq 0$?
   (b) For which value(s) of z are $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ linearly dependent?

Section 3.5

9. Consider the following set:

$$W = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -1 \\ -7 \end{pmatrix} \right\}.$$

   (a) (MP) Find a basis for $\operatorname{Span} W$.
   (b) (BH) Find $\dim(\operatorname{Span} W)$.

10. (BH) Find a basis for

$$\operatorname{Span} \left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \right\}.$$