MATH 342-010 Differential Equations with Linear Algebra II
Prof. D. A. Edwards Due: Nov. 14, 2007

Homework Set 8

Read Leon, sections 5.2 and 5.3.

Section L5.2

1. page 233, number 6
2. page 234, number 13
3. page 234, number 17
4. Consider the following matrix:

$$B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & -1 \end{pmatrix}.$$

   (a) Find bases for $\mathcal{N}(B)$, $R(B)$, $\mathcal{N}(B^T)$, and $R(B^T)$.
   (b) Show explicitly that $R(B^T) \perp \mathcal{N}(B)$ and $R(B) \perp \mathcal{N}(B^T)$.
   (c) Decompose the vector $\mathbf{x} = (1, 1, 1)^T$ into components in $R(B)$ and $\mathcal{N}(B^T)$.

5. Consider the following matrix:

$$C = \begin{pmatrix} 1 & -2 & 3 & -4 \\ -1 & 3 & -2 & 5 \\ 1 & -1 & 4 & -3 \end{pmatrix}.$$

   (a) Find bases for $\mathcal{N}(C)$, $R(C)$, $\mathcal{N}(C^T)$, and $R(C^T)$.
   (b) Show explicitly that $R(C^T) \perp \mathcal{N}(C)$ and $R(C) \perp \mathcal{N}(C^T)$.
   (c) Decompose the vector $\mathbf{x} = (2, 0, -2)^T$ into components in $R(C)$ and $\mathcal{N}(C^T)$.

Section L5.3

6. Consider the following system:

$$A\mathbf{x} = \mathbf{b}, \qquad A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & \kappa \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 6 \\ \alpha \end{pmatrix}. \tag{8.1}$$

   (a) If $\kappa \neq 6$, (8.1) has a unique solution. Find it.
   (b) If $\kappa = 6$ and α has a particular value, (8.1) has a solution. Find the special value of α and the corresponding solution(s) of (8.1).
   (c) Find the least-squares solution(s) of (8.1) if $\kappa = 6$ and α does not equal the special value in (b).

7. Determine the least-squares *parabola* for the data points

$$(1, 5),\ (2, 2),\ (3, 3),\ (4, 8).$$

8. A political scientist hypothesizes the following law for presidential popularity:

$$P = P_0 + P_u u + P_r r,$$

where P is the popularity, u is the unemployment rate, and r is the inflation rate. Suppose that the following data is available:

Popularity P	Unemployment rate u	Inflation rate r
52	6	4
41	5	6
40	6	8
54	4	10

(a) Calculate P_0, P_u, and P_r for this system.
(b) Intepret the sign of P_u and P_r sociologically.

9. Suppose that a certain ODE/linear algebra professor has been strapped to a medieval rack.
Under a force of 1 ton, his length is stretched to 195 cm.
Under a force of 2 tons, his length is stretched to 320 cm.
Under a force of 4 tons, his length is stretched to 360 cm.
If we consider his stretched corpse to be a spring, then according to Hooke's Law, his length L under a force F would be given by

$$L = a + bF,$$

where a and b are constants.

(a) Given the data above, find a and b.
(b) Using Hooke's Law, what was the professor's original height?

10. In economics, the supply and demand of a particular item at a particular cost can often be modeled relatively simply. For instance, a typical demand curve is often given by

$$D(C) = \beta_0 + \frac{\beta_1}{C}, \qquad \beta_1 > 0, \quad C > 0,$$

where $D(C)$ is the number of units demanded if the cost per unit is C.

(a) Calculate

$$\lim_{C \to 0} D(C)$$

and interpret your result.

(b) Calculate

$$\lim_{C \to \infty} D(C).$$

What does it mean for $\beta_0 \neq 0$? (It is theorized that certain addictive substances, like illegal drugs, have $\beta_0 \neq 0$.)

In addition, a typical supply curve is often given by

$$S(C) = \alpha_0 + \alpha_1 C, \qquad C \geq 0,$$

where $S(C)$ is the number of units on hand if the cost per unit is C.

(c) Calculate

$$\lim_{C \to 0} S(C).$$

What does it mean for $\alpha_0 \neq 0$? (Many easy-to-make products have $\alpha_0 \neq 0$.)

Now suppose that the following data have been collected regarding a certain product:

Cost per unit C	Units demanded D	Units on hand S
1/3	7	—
1/2	2	—
1	3	1
2	—	8/3
5	—	10/3

(d) Calculate β_0 and β_1 for this system.

(e) Calculate α_0 and α_1 for this system.

(f) Using your answers to (d) and (e), estimate the *equilibrium cost*, which is the cost at which $D(C) = S(C)$.