Updates

1. Exam II will be administered on Monday, Oct. 29. You will need to bring your own blue book.
2. The final exam will be administered on Thursday, Dec. 13 from 10:30–12:30. You will need to bring your own blue book.

Homework Set 7

Read Leon, sections 5.1 and 5.4.

Section L5.1

1. page 224, number 11
2. page 224, number 13
3. Let l_1 be the line $y = m_1 x + b_1$, where $m_1 \neq 0$. Let l be the line $y = mx + b$, where $m \neq 0$. Show that $l \perp l_1$ if and only if $m = -1/m_1$.
4. Consider the vectors

$$\mathbf{v} = (1, 3, -4, 2)^T, \qquad \mathbf{w} = (0, 2, 4, -1)^T.$$

   (a) Find the scalar projection of $\mathbf{v}$ onto $\mathbf{w}$.
   (b) Find the vector projection of $\mathbf{w}$ onto $\mathbf{v}$.
   (c) Show that the component of $\mathbf{w}$ orthogonal to $\mathbf{v}$ is given by

$$\left(\frac{2}{5}, \frac{16}{5}, \frac{12}{5}, -\frac{1}{5}\right)^T.$$

   (d) Verify that your answers to (b) and (c) are orthogonal.
5. Consider a cube with two corners (not on the same face) at $(-1, -1, 1)^T$ and $(1, 1, -1)^T$.
   (a) Show that the angle between the diagonal of the cube and one of its edges is given by $\cos^{-1}(3^{-1/2})$.
   (b) Find the angle between a diagonal of the cube and a diagonal of one of its faces.

Section L5.4

6. page 254, number 26. Here $C[a, b]$ is the vector space of all functions that are continuous for $x \in [a, b]$. Be sure to explain your reasoning.

7. Consider the following vector product on the vector space $C[-\pi, \pi]$, which we used in class:

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)\,dx.$$

   (a) Calculate $\langle e^x, e^{-x} \rangle$.
   (b) Find the projection of e^{-x} onto $\sin x$ and the component of e^{-x} orthogonal to $\sin x$.
   (c) Explain why e^{x^2} and $\sin x$ are orthogonal. (*Hint: You should not try to calculate the integral directly.*)

8. Consider the following relation:

$$||\mathbf{x}||_\infty = \max_i |x_i|, \qquad \mathbf{x} \in \mathcal{R}^n.$$

   Show that $||\mathbf{x}||_\infty$ is a vector norm by verifying the properties shown in class.

9. Show that

$$\lim_{p \to \infty} ||\mathbf{x}||_p = ||\mathbf{x}||_\infty,$$

   where

$$||\mathbf{x}||_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \qquad \mathbf{x} \in \mathcal{R}^n$$

   is the p-norm defined in class.

10. Consider the following vectors:

$$\mathbf{x} = (-1, 0, 3)^T, \qquad \mathbf{y} = (3, -2, -1)^T.$$

   (a) Calculate $||\mathbf{x} - \mathbf{y}||_1$, $||\mathbf{x} - \mathbf{y}||_2$, and $||\mathbf{x} - \mathbf{y}||_\infty$.
   (b) In which norm are the two vectors closest together? In which are they furthest apart?