Homework Set 3 (Revised)

Read Leon, section 4.3. Read Edwards and Penney, section 7.1.

Section L4.3

1. page 206, number 8
2. page 206, number 14. (Thus, similar matrices have the same eigenvalues.)
3. Consider the following matrices:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -1 \\ -6 & 4 \end{pmatrix}, \qquad S = \begin{pmatrix} 1 & s_{12} \\ s_{21} & -2 \end{pmatrix}.$$

   Find s_{12} and s_{21} so that A and B will be similar matrices as defined on page 202 (which may be different from the one I gave in class). (*Hint: How can you do this without calculating S^{-1}?*)

4. Let

$$S = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}, \qquad T = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\}$$

   be bases for $\mathcal{R}^3$ and $\mathcal{R}^2$, and let

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \end{pmatrix}$$

   be the matrix representation of some operator $L : \mathcal{R}^3 \to \mathcal{R}^2$ with respect to S and T. Find the matrix representation for L in the standard bases.

5. Consider the following linear transformation on P_2:

$$L(p(t)) = t^2\ddot{p} + \dot{p}.$$

   (a) Find the matrix A representing L with respect to the standard basis $E : [t^2, t, 1]$.
   (b) Write the transition matrix T representing the change of coordinates from the basis $F : [t^2 + t, t - 1, 2]$ to the standard basis E.
   (c) Write the matrix B representing L with respect to the basis F.
   (d) Explain (in terms of changing from one basis to another) why $B = T^{-1}AT$, and verify it by direct calculation.

Section E7.1

6. For what range of s will the Laplace transforms of the following functions exist?
   (a) $\sin \omega t$ (ω is a real constant)
   (b) $\cos \omega t$
   (c) $e^{3\sqrt{t}}$
   (d) $e^{\omega t^2}$
7. Calculate the Laplace transform of the following functions. Use the *definition*, not the table.
   (a) $\cos \omega t$
   (b) $\sin \omega t$. In this case,

$$\mathcal{L}\{\sin \omega t\} = \int_0^\infty e^{-st} \sin \omega t \, dt. \tag{3.1}$$

   (c) $t \sin \omega t$. (*Hint: Differentiate (3.1) with respect to s.*)
8. Calculate the Laplace transform of the following functions. (*Hint: Use trigonometric identities and your answers to #7.*)
   (a) $3 \sin^2 2t - 4$
   (b) $4 \cos \left(t - \frac{\pi}{3}\right)$
9. Consider the following function:

$$f(t) = \begin{cases} t, & 0 \le t < 1, \\ 0, & t \ge 1. \end{cases}$$

   (a) Sketch this function.
   (b) Compute the Laplace transform of f.
10. Show that

$$\mathcal{L}\{\log t\} = -\frac{\log s}{s} - \frac{\gamma}{s}, \qquad \gamma = -\int_0^\infty e^{-u} \log u \, du.$$

   γ is called *Euler's constant*. (*Hint: Use the definition of γ to motivate an appropriate substitution in the integrand.*)