Phase Plane: Real Eigenvalues

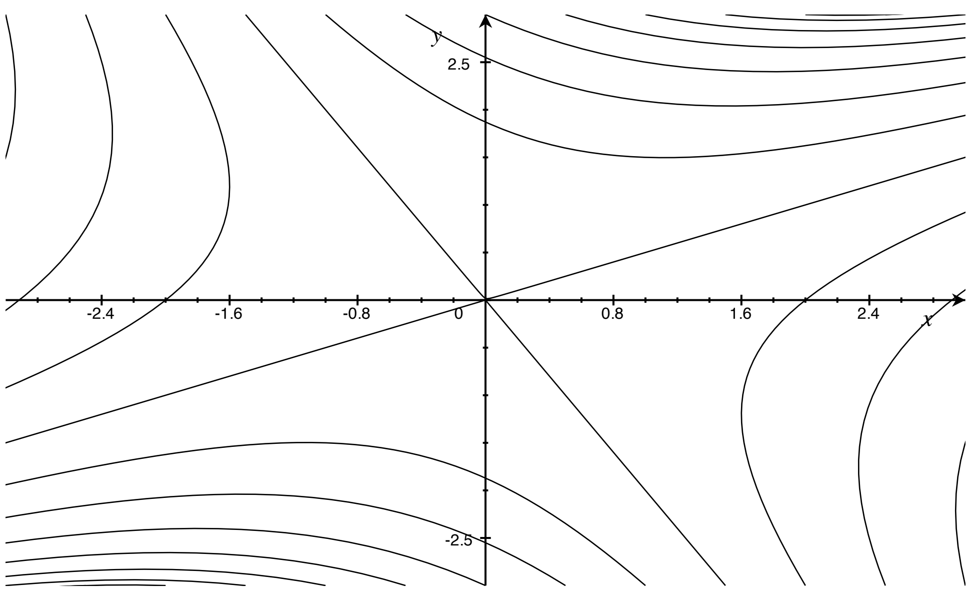
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \mathbf{x}, \tag{1}$$

the solution is

$$\mathbf{x} = c_1 e^{5t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{-5t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Since we have one positive and one negative eigenvalue, we have a saddle point, as shown below. Note the straight lines corresponding to the eigenvectors.



Phase plane of (1).

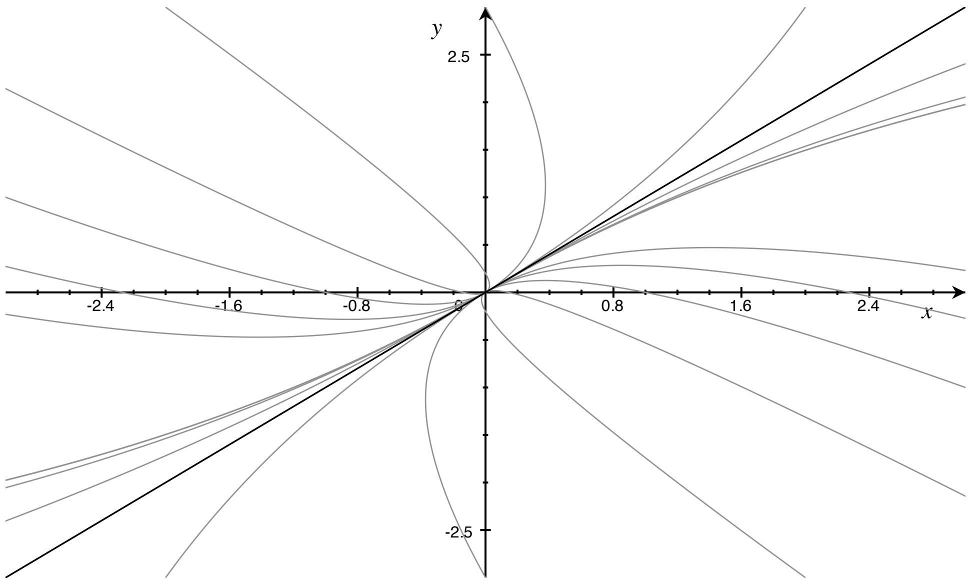
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix} \mathbf{x}, \tag{2}$$

the solution is

$$\mathbf{x} = c_1 e^{-4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Since we have two negative eigenvalues, we have a stable node, as shown below.



Phase plane of (2).