

Facts About Invertible Matrices

Let $A \in \mathcal{R}^{n \times n}$. Then the following statements are *equivalent*. By that I mean that if one of them is true, they are all true. If one of them is false, they are all false.

Basic Facts

1. A is invertible.
2. $\det A \neq 0$.
3. 0 is not an eigenvalue for A .

Solutions of Linear Systems

4. $A\mathbf{x} = \mathbf{b}$ has exactly one solution for every $\mathbf{b} \in \mathcal{R}^n$ (no free variables).
5. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

Row Reduction

6. A can be row reduced to I .
7. A is a product of elementary matrices.
8. A has n pivot columns.

Row, Column, and Null Spaces

9. The columns of A are linearly independent.
10. $\text{col } A = \mathcal{R}^n$, and the columns of A form a basis for \mathcal{R}^n .
11. The rows of A are linearly independent.
12. $\text{row } A = \mathcal{R}^n$, and the rows of A form a basis for \mathcal{R}^n .
13. $\text{rank } A = n$.
14. $\mathcal{N}(A) = \{\mathbf{0}\}$.
15. $\dim \mathcal{N}(A) = 0$.