

Phase Plane: Complex Eigenvalues

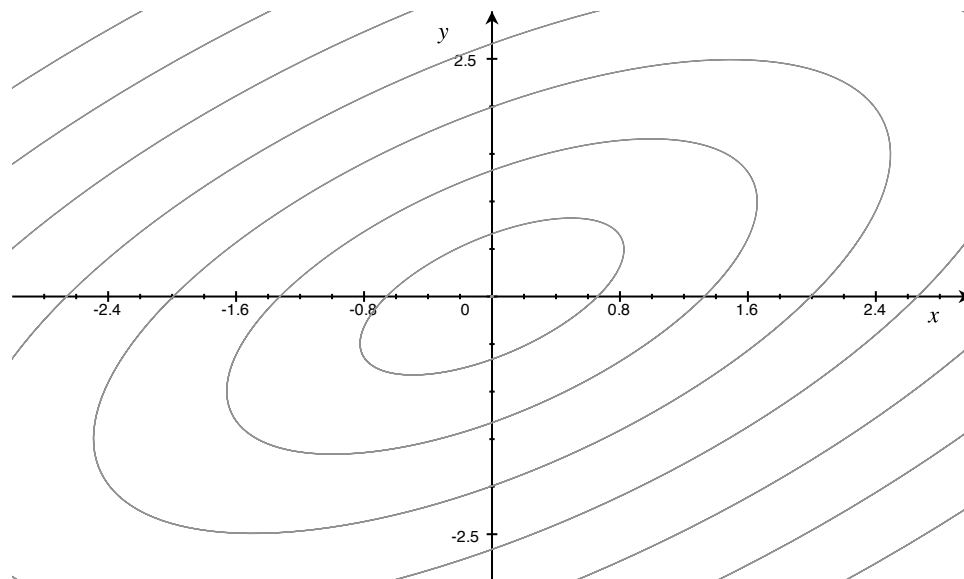
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} 3 & -5 \\ 5 & -3 \end{pmatrix} \mathbf{x}, \quad (1)$$

the solution is

$$\mathbf{x} = c_1 \begin{pmatrix} 5 \cos 4t \\ 3 \cos 4t + 4 \sin 4t \end{pmatrix} + c_2 \begin{pmatrix} 5 \sin 4t \\ 3 \sin 4t - 4 \cos 4t \end{pmatrix}.$$

Since there are no real eigenvectors, all trajectories spin about the origin. Since the real part of the eigenvalues is zero, the origin is a center, as shown below:



Phase plane of (1).

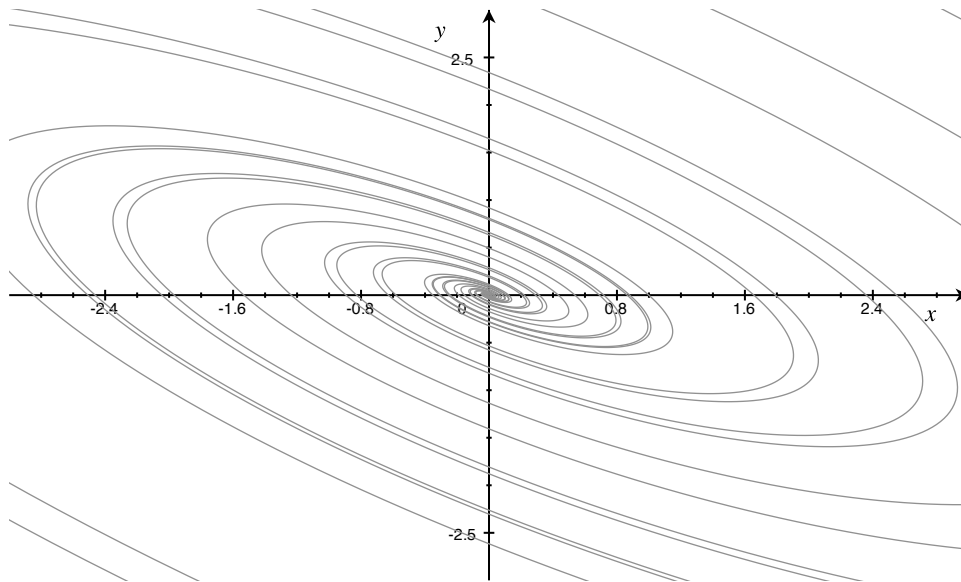
For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -2 & -6 \\ 3 & 4 \end{pmatrix} \mathbf{x}, \quad (2)$$

the solution is

$$\mathbf{x} = c_1 e^t \begin{pmatrix} -\cos 3t - \sin 3t \\ \cos 3t \end{pmatrix} + c_2 e^t \begin{pmatrix} \cos 3t - \sin 3t \\ \sin 3t \end{pmatrix}.$$

Since the real part of the eigenvalues is positive, we have an unstable spiral, as shown below.



Phase plane of (2).