Series Solutions

Consider the equation

$$y'' - y = 0.$$

The solution y_1 with $y_1(0) = 1$, $y_1'(0) = 0$ is $\cosh x$, which we showed had a series solution given by

$$y_1 = \cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}.$$

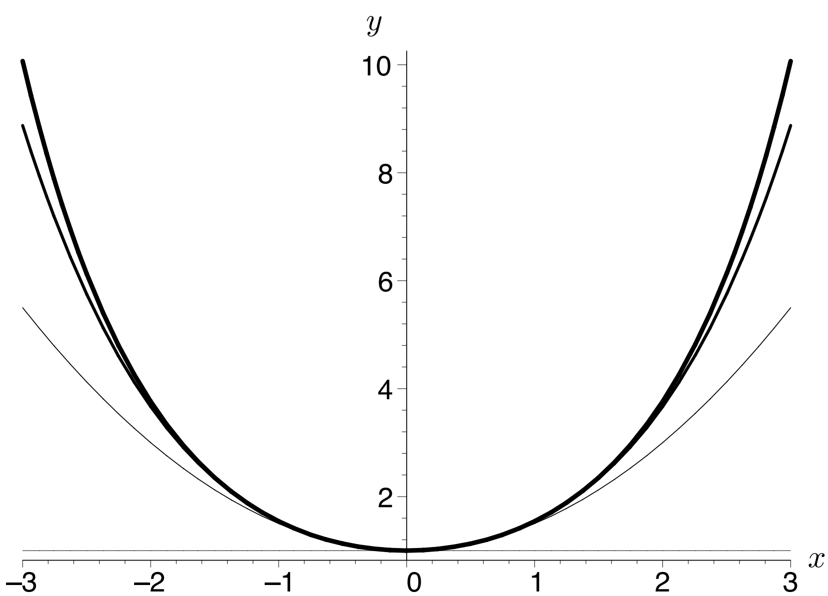
The first three approximations to the series are then given by

$$y_1 \approx 1, \tag{1.1}$$

$$y_1 \approx 1 + \frac{x^2}{2}, \tag{1.2}$$

$$y_1 \approx 1 + \frac{x^2}{2} + \frac{x^4}{24}. \tag{1.3}$$

These approximations are graphed below. Note that with each increasing term, the range of x for which the polynomial is a good approximation widens.



In increasing order of thickness: Polynomial approximations (1.1), (1.2), (1.3), and $\cosh x$ *vs.* x for $x \in [-3, 3]$.

The solution y_2 with $y_2(0) = 1$, $y_2'(0) = 0$ is $\sinh x$, which we showed had a series solution given by

$$y_2 = \sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}.$$

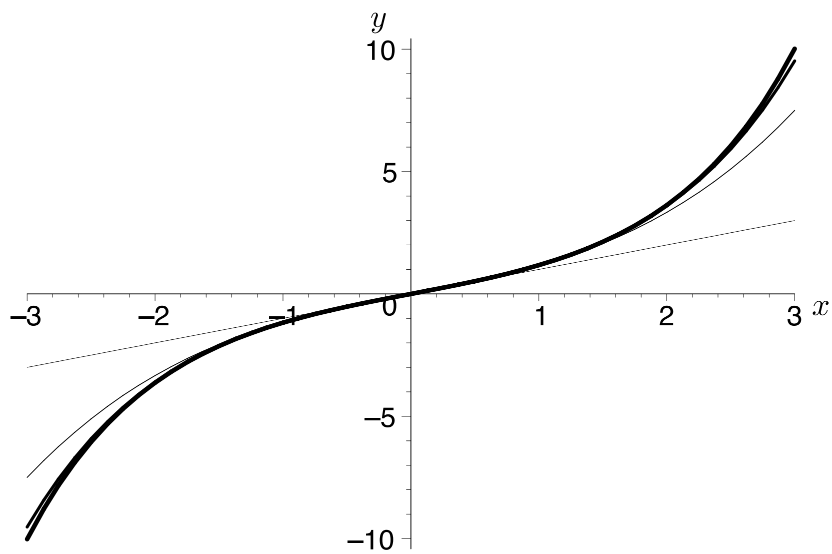
The first three approximations to the series are then given by

$$y_2 \approx x, \tag{2.1}$$

$$y_2 \approx x + \frac{x^3}{6}, \tag{2.2}$$

$$y_2 \approx 1 + \frac{x^3}{6} + \frac{x^5}{120}. \tag{2.3}$$

These approximations are graphed below. Note that with each increasing term, the range of x for which the polynomial is a good approximation widens.



In increasing order of thickness: Polynomial approximations (2.1), (2.2), (2.3), and $\sinh x$ *vs.* x for $x \in [-3, 3]$.