

## Writing Assignment

As graduate students in mathematics, it is reasonable to assume that at some point in your career you will need to communicate mathematical reasoning in writing to someone, whether it be for a paper in a published journal, a Ph.D. dissertation or master's thesis, or a "white paper" for one's employer. Though each homework assignment in and of itself is a written assignment where you (should) explain your mathematical reasoning in writing, it is much less structured than a standard paper. In addition, your audience (the grader and I) is one which is very familiar with the material at hand. Therefore, you can be justified in skipping some steps in your analysis.

In order to give you some practice in writing, I would like you to write up *thoroughly* one of the problems given below, or one of your own choosing (provided you clear it with me first). I want you to work on an applied problem rather than a proof. The level of mathematical knowledge involved will be that which was presented in chapter 5. I want you to gain practice in expressing your ideas, rather than testing your knowledge of difficult mathematical concepts. This assignment, as opposed to the other ones, is **MANDATORY**. However, since it is substantially different from the other assignments, I can not yet ascertain how it will fit into the grading pattern.

The assignment consists of three parts:

1. Since the topics listed below are very broad, you should talk to me sometime about your particular project before beginning it in earnest. That way, I may be able to assist you in avoiding mathematical pitfalls which would needlessly complicate the project. I cannot stress enough that I want the mathematics to be relatively simple; I am looking for careful exposition of your ideas.
2. The written paper, which should consist of
  - (a) an introduction to the physical problem, indicating the assumptions you made to simplify it into a linear system.
  - (b) a careful exposition of how you solved the problem. This should be written for an audience which has some knowledge of the mathematical and background material, but not an extensive one. Pretend you are teaching the class. You are encouraged to use the computer to solve the problems, but you should explain what you are doing (and why) at each step.
  - (c) an analysis of your mathematical results, and what they reveal about the real-world problem. Was your model too simple to capture the real-world behavior? How could you extend your model to make it more realistic? Does your solution have any useful predictive capabilities?

Though it would be nice if the paper were typed, it is not required.

3. Once the papers have been turned in and I have had a chance to examine them, I would like for us to discuss them over refreshments, with each of you sharing the

problem on which you worked, how you attacked it mathematically, what you learned from the experience, and how this assignment could be modified to make it more successful.

### Sample Problems

1. **ECONOMICS.** Consider three or four economic indicators/properties which you think are related. Hypothesize some relationships between the properties and formulate the problem as a *discrete* process. (To simplify matters, you should only think about systems with one- or two-step memories.) Write the problem as a matrix system, and then solve and interpret the eigenvalues and eigenvectors. For example:
  - (a) Suppose that the following variables are linearly related: the number of housing starts in the last two months, the prime rate in the last two months, consumer confidence levels in the last two months, and the unemployment rate in the last two months. Write the future values of these variables in terms of the past values, and solve the accompanying discrete system.
  - (b) Suppose that the probability of the following variables are linearly related: the stock market rising or falling in a particular month, the unemployment rate rising or falling in a particular month, and factory output rising or falling in a particular month. Write the system as a Markov-like process, and solve for the steady state(s).
2. **BIOLOGY.** Consider a system of four or five interrelated species. Postulate the dependence of their population growth rates as a function of their populations. Solve the associated system of ordinary differential equations.
 

*Example.* You wish to grow a bacteria culture used for medicinal purposes in a Petri dish. In any initial sample, there will also be some nonuseful bacteria whose population we wish to limit. You must provide food, which will cause both bacteria to grow (but perhaps at different rates), and you can provide a level of germicide, which will cause the nonuseful bacteria to die (but will also kill some of the useful bacteria). Solve the accompanying system.
3. **PHYSICS.** Consider a system of masses coupled by springs of various spring constants. Find the normal modes of the system and explain their significance.
4. **STOCHASTIC PROCESSES.** Model a diffusion or other process by a random walk involving several variables. Solve the accompanying discrete system for the steady state(s).
5. **ELECTRICAL ENGINEERING.** Model a system of interconnected circuit loops as a system of ordinary differential equations. Solve the associated system with both AC and DC sources, and interpret your answers.