

Homework Set 4

Read section 5.4 of the text.

Section 5.4

1. Verify the properties of e^{tA} given in class, namely that
 - (a) (1 point) $e^{tA}e^{sA} = e^{(t+s)A}$
 - (b) (1 point) $e^{tA}e^{tB} = e^{t(A+B)}$ iff A and B commute
 - (c) (1 point) $d(e^{tA})/dt = Ae^{tA}$
 - (d) (1 point) $(e^{tA})^T = e^{tA^T}$
2. (2 points) page 286, exercise 5.4.2
3. (4 points) page 286, exercise 5.4.8
4. Consider the following system:

$$\ddot{\mathbf{x}} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} a \\ 2b \end{pmatrix}, \quad \dot{\mathbf{x}}(0) = \begin{pmatrix} c - d \\ 2d \end{pmatrix}$$

- (a) (1 point) Is this system stable, neutrally stable, or unstable?
- (b) (2 points) Solve the system for \mathbf{x} .
- (c) (2 points) Consider your solution as $t \rightarrow \infty$. Does it diverge/converge exponentially? algebraically? Reconcile your answers to this part with your answers to part (a).
- (d) (1 point) Are there any values of (a, b, c, d) for which the system remains bounded as $t \rightarrow \infty$?