# A complete span of $\mathcal{H}(4,4)$ admitting $PSL_2(11)$ and related structures

**A. Cossidente, G. L. Ebert*, and G. Marino**

**Abstract**

We construct a complete 11–span of $\mathcal{H}(4,4)$ admitting the group $\mathrm{PSL}_2(11)$. This span turns out to be associated with the unique Hadamard design $\mathcal{H}_{11}$ and the so-called Petersen design.

**Keywords:** Hermitian variety, complete span, maximal subgroup

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# 1 Introduction

A *spread* of a finite polar space $\mathcal{P}$ is a set of mutually skew subspaces of maximum dimension (*generators*) that cover $\mathcal{P}$. Any set of mutually skew generators is called a *partial spread* or *span*. If a spread does not exist, it is natural to ask how large a span can be. A span is said to be *complete* if it is maximal with respect to set–theoretic inclusion. An equally natural question to ask is the following: what is the smallest possible size for a complete span? In this paper the polar space involved is the Hermitian variety $\mathcal{H}(4, q^2)$ of $PG(4, q^2)$. It was shown in [2] that $\mathcal{H}(4, 4)$ has no spread, but the existence of a spread of $\mathcal{H}(4, q^2)$, $q > 2$, is still an open problem. On the other hand, in [6] it has been shown that a complete span of $\mathcal{H}(4, q^2)$ must have size at least $q^3 + q\sqrt{q} - 1/2q - 3/8\sqrt{q} + 7/8$. Hence, when $q = 2$, a complete span of $\mathcal{H}(4, 4)$ must have at least 11 lines.

Exploring the geometry of orbits of $\mathrm{PSL}_2(11)$ on generators of $\mathcal{H}(4, 4)$, we construct an example of complete 11–span of $\mathcal{H}(4, 4)$ admitting the linear group $\mathrm{PSL}_2(11)$. This 11–span is associated with another complete 11–span (its companion), covering the same point set. We found that these two 11–spans give rise to the unique Hadamard design $\mathcal{H}_{11}$ in a very natural way. We also show that the Petersen design can be constructed from the orbits of $\mathrm{PSL}_2(11)$ on generators.

# 2 On the group $\mathrm{PSL}_2(11)$

From [7] one sees that $G = \mathrm{PSL}_2(11)$ has a 5-dimensional representation over $K = \mathrm{GF}(4)$ in which $G$ fixes a non–degenerate Hermitian form on the underlying vector space $V$. Hence we see that $G < \mathrm{PSU}_5(4)$. It is this embedding that we wish to exploit in the following constructions.

Using MAGMA [3], we first construct the unique subgroup $G$ of order 660 in $\mathrm{PSU}_5(4)$, where necessarily $G \cong \mathrm{PSL}_2(11)$ as above. The equation used by MAGMA for the invariant Hermitian variety $\mathcal{H}(4, 4)$ is $x_0x_4^2 + x_1x_3^2 + x_2^3 + x_3x_1^2 + x_4x_0^2 = 0$. The group $G$ has five orbits on the generators of $\mathcal{H}(4, 4)$, say $L_1, \ldots, L_5$, with $|L_1| = |L_2| = 11$, $|L_3| = |L_4| = 55$, and $|L_5| = 165$. Computer computations show that the generators of $L_1$ form an 11–span of $\mathcal{H}(4, 4)$, as do the generators of $L_2$. These two spans cover the same set of 55 points on $\mathcal{H}(4, 4)$, and any other generator necessarily meets this set in at least one point. Hence $L_1$ and $L_2$ are (companion) complete spans of $\mathcal{H}(4,4)$, meeting the lower bound mentioned above. Letting $\omega$ denote a primitive element of $K = \mathrm{GF}(4)$, these complete spans are the following:

$$\begin{aligned}
L_1 \;=\;& \{\{(1,\omega,\omega,0,\omega^2),\ (1,\omega^2,\omega,1,0),\ (1,1,\omega,\omega^2,1),\ (0,1,0,1,\omega^2),\\
& (1,0,\omega,\omega,\omega)\},\ \{(1,1,\omega^2,\omega,1),\ (0,1,0,1,\omega),\ (1,\omega,\omega^2,1,0),\\
& (1,0,\omega^2,\omega^2,\omega^2),\ (1,\omega^2,\omega^2,0,\omega)\},\ \{(0,1,\omega^2,\omega^2,1),\ (1,1,\omega,0,\omega),\\
& (1,0,1,\omega^2,\omega^2),\ (1,\omega^2,\omega^2,1,0),\ (1,\omega,0,\omega,1)\},\ \{(1,\omega^2,\omega^2,\omega^2,\omega^2),\\
& (0,1,\omega^2,\omega^2,\omega),\ (1,1,\omega,\omega,0),\ (1,\omega,0,0,1),\ (1,0,1,1,\omega)\},\\
& \{(1,\omega,\omega,1,1),(1,\omega^2,1,\omega^2,\omega^2),\ (1,0,\omega^2,\omega,\omega),\ (0,1,\omega^2,\omega,\omega),\\
& (1,1,0,0,0)\},\ \{(1,1,0,\omega^2,\omega),\ (1,\omega^2,\omega^2,\omega,0),\ (1,\omega,1,1,1),\\
& (0,1,\omega,\omega^2,1),\ (1,0,\omega,0,\omega^2)\},\ \{(0,1,\omega^2,\omega,1),\ (1,\omega^2,\omega,0,\omega^2),\\
& (1,0,0,1,0),\ (1,\omega,1,\omega,\omega),\ (1,1,\omega^2,\omega^2,1)\},\ \{(1,0,\omega,1,\omega),\\
& (1,1,1,\omega,1),\ (1,\omega,\omega^2,0,\omega^2),\ (0,1,\omega^2,\omega^2,\omega^2),\ (1,\omega^2,0,\omega^2,0)\},\\
& \{(1,1,1,0,\omega),\ (1,\omega^2,0,1,\omega),\ (1,\omega,\omega^2,\omega,\omega),\ (1,0,\omega,\omega^2,\omega),\\
& (0,1,\omega^2,\omega^2,0)\},\ \{(0,1,\omega,\omega,\omega),\ (0,1,\omega,\omega,\omega^2),\ (0,0,0,0,1),\\
& (0,1,\omega,\omega,0),\ (0,1,\omega,\omega,1)\},\ \{(1,\omega^2,1,0,\omega^2),\ (1,1,\omega,1,\omega),\\
& (1,\omega,\omega^2,\omega^2,0),\ (0,1,\omega,\omega^2,\omega^2),\ (1,0,0,\omega,1)\}\},
\end{aligned}$$

$$\begin{aligned}
L_2 \;=\;& \{\{(1,0,\omega^2,\omega,\omega),\ (0,1,0,1,\omega^2),\ (1,\omega,\omega^2,0,\omega^2),\ (1,\omega^2,\omega^2,1,0),\\
& (1,1,\omega^2,\omega^2,1)\},\ \{(0,1,\omega^2,\omega^2,1),\ (0,1,\omega^2,\omega^2,\omega),\ (0,0,0,0,1),\\
& (0,1,\omega^2,\omega^2,\omega^2),\ (0,1,\omega^2,\omega^2,0)\},\ \{(1,\omega^2,1,\omega^2,\omega^2),\ (1,\omega,\omega^2,1,0),\\
& (0,1,\omega,\omega,\omega^2),\ (1,1,\omega,0,\omega),\ (1,0,0,\omega,1)\},\ \{(1,\omega^2,\omega^2,\omega^2,\omega^2),\\
& (0,1,\omega,\omega,\omega),\ (1,\omega,1,1,1),\ (1,0,\omega,\omega,\omega),\ (1,1,0,0,0)\},\\
& \{(1,1,\omega^2,\omega,1),\ (1,\omega,\omega,0,\omega^2),\ (0,1,\omega,\omega^2,\omega^2),\ (1,0,1,1,\omega),\\
& (1,\omega^2,0,\omega^2,0)\},\ \{(1,\omega,\omega,1,1),\ (0,1,0,1,\omega),\ (1,1,\omega,\omega,0),\\
& (1,\omega^2,\omega,0,\omega^2),\ (1,0,\omega,\omega^2,\omega)\},\ \{(1,\omega,\omega^2,\omega^2,0),\ (1,1,1,\omega,1),\\
& (0,1,\omega^2,\omega,\omega),\ (1,\omega^2,0,1,\omega),\ (1,0,\omega,0,\omega^2)\},\ \{(1,\omega^2,1,0,\omega^2),\\
& (1,1,\omega,\omega^2,1),\ (1,0,0,1,0),\ (1,\omega,\omega^2,\omega,\omega),\ (0,1,\omega,\omega,1)\},\\
& \{(1,0,\omega,1,\omega),\ (1,1,0,\omega^2,\omega),\ (0,1,\omega,\omega,0),\ (1,\omega^2,\omega^2,0,\omega),\\
& (1,\omega,1,\omega,\omega)\},\ \{(1,1,\omega,1,\omega),\ (0,1,\omega^2,\omega,1),\ (1,\omega,0,0,1),\\
& (1,0,1,\omega^2,\omega^2),\ (1,\omega^2,\omega^2,\omega,0)\},\ \{(1,1,1,0,\omega),\ (1,\omega^2,\omega,1,0),\\
& (1,0,\omega^2,\omega^2,\omega^2),\ (0,1,\omega,\omega^2,1),\ (1,\omega,0,\omega,1)\}\}.
\end{aligned}$$

Now every line in $L_2$ meets exactly 5 lines of $L_1$ (in one point each), and hence we determine 11 (distinct) subsets of $L_1$, each of size 5. Moreover, direct computations show that each pair of lines in $L_1$ lies in exactly 2 of these subsets. That is, we have constructed the $2-(11,5,2)$ biplane. This, of course, is the famous Hadamard design $\mathcal{H}_{11}$. The complementary design is a $2-(11,6,3)$ BIBD.

One of the $G$-orbits on generators of size 55, say $L_4$, has the property that each line of $L_4$ meets exactly 3 lines of $L_1$, and hence we so determine 55 distinct subsets of $L_1$, each of size 3. More direct computations show that each pair of lines in $L_1$ lies in exactly 3 such subsets, and hence we have a $2-(11,3,3)$ BIBD. This design is known as the *Petersen design* [1].

From a group–theoretic point of view, the Petersen design can be described as follows. There are 55 involutions in $\mathrm{PSL}_2(11)$, and each of them has as axis a generator in $L_4$ [4], and each of them fixes 3 lines in $L_1$.

We also make the following group-theoretic observation.

**Proposition 2.1.** *$G$ is a maximal subgroup of* $\mathrm{PSU}_5(4)$.

**Proof**. The group $G$ has just two orbits on points of $\mathcal{H}(4,4)$, one of size 55 and one of size 110. Let $F$ be any subgroup of $\mathrm{PSU}_5(4)$ such that

$$G < F \leq \mathrm{PSU}_5(4).$$

Since $G$ is the full stabilizer of $L_1$ in $\mathrm{PSU}_5(4)$, $F$ must act transitively on points of $\mathcal{H}(4,4)$. From [5, Cor. 5.12 ] necessarily $F = \mathrm{PSU}_5(4)$, and hence $G$ is maximal in $\mathrm{PSU}_5(4)$. □

# 3 $\mathrm{PSL}_2(11)$ as a subgroup of $\mathrm{PSU}_6(4)$

Embedding $\mathcal{H}(4,4)$ in $\mathcal{H}(5,4)$, we now look at the action of $\mathrm{PSL}_2(11)$ on generators of $\mathcal{H}(5,4)$. We again use MAGMA to find the unique subgroup $G$ of order 660 in $\mathrm{PSU}_6(4)$, where necessarily $G \cong \mathrm{PSL}_2(11)$. The equation used by MAGMA for $\mathcal{H}(5,4)$ is $x_0x_5^2 + x_1x_4^2 + x_2x_3^2 + x_3x_2^2 + x_4x_1^2 + x_5x_0^2 = 0$. Computer computations show that $G$ has 15 orbits on the generators of $\mathcal{H}(5,4)$, of which 6 have size 11. The 11 planes in any such orbit have the property that any two of them meet in exactly one point. Moreover, we can pair off these 6 orbits in such a way that, upon taking the union of each pair of orbits, we obtain 3 sets of 22 generators of $\mathcal{H}(5,4)$ with this same intersection property. That is, any two planes from the same set of 22 generators will meet in exactly one point. Let $\{P_1, P_4\}$ be such a pair of orbits, and let $P = P_1 \cup P_4$ denote the union of these orbits. Thus $P$ consists of 22 generators of $\mathcal{H}(5,4)$, any two of which meet in exactly one point.

The number of points covered by the generators in $P_1$ is 176, as is true for the generators in $P_4$. The intersection $\mathcal{Q}$ of these two point sets is a collection of 121 points on $\mathcal{H}(5,4)$ with the property that each such point lies on exactly one generator from the orbit $P_1$ and lies on exactly one generator from the orbit $P_4$. Thus we get a type of "grid" induced on this set of points in $\mathcal{H}(5,4)$.

Now let $\pi_1$ denote some plane in $P_1$. MAGMA computations show that the stabilizer $H_1$ of $\pi_1$ in $G$ is isomorphic to the alternating group $A_5$, and $H_1$ has three point orbits on $\pi_1$. These point orbits have sizes 5, 6, and 10, with the point orbit $O_1$ of size 6 being a hyperoval in $\pi_1$. More direct computations show that the orbit of $O_1$ under $G$ yields a collection of 11 disjoint hyperovals covering 66 points of $\mathcal{Q}$. Each plane of $P_1$ contains one of these hyperovals. Let $\mathcal{O}_1$ denote this collection of disjoint hyperovals. Similarly, starting with some plane $\pi_4$ in $P_4$, one obtains another collection $\mathcal{O}_4$ of 11 disjoint hyperovals, one contained in each plane of $P_4$. The hyperoval collections follow:

$$\begin{aligned}\mathcal{O}_1 \;=\; & \{\{(0,0,1,0,\omega,1),\ (1,0,1,1,1,1),\ (0,1,0,\omega^2,1,\omega),\ (1,1,\omega,\omega,1,0),\ (1,\omega^2,0,\omega^2,0,1),\\ & (1,\omega,\omega^2,0,0,0)\},\ \{(1,1,0,0,\omega,\omega),\ (0,1,0,\omega,0,\omega^2),\ (1,\omega^2,0,1,\omega,\omega^2),\\ & (1,0,0,1,\omega,1),\ (1,\omega,0,0,\omega,0),\ (0,0,0,1,0,0)\},\ \{(1,1,1,1,0,0),\ (0,1,0,0,0,0),\\ & (1,\omega^2,1,\omega^2,0,\omega),\ (0,0,0,1,0,1),\ (1,\omega,1,0,0,1),\ (1,0,1,\omega,0,\omega^2)\},\\ & \{(0,1,\omega,1,\omega^2,0),\ (1,0,0,0,0,1),\ (1,\omega^2,1,\omega,1,1),\ (0,0,0,1,\omega^2,0),\ (1,\omega,\omega^2,0,0,1),\\ & (1,1,\omega,\omega,1,1)\},\ \{(1,\omega^2,1,1,\omega,\omega^2),\ (0,0,0,0,1,\omega),\ (1,\omega^2,\omega^2,\omega^2,0,0),\\ & (0,0,1,1,0,0),\ (1,\omega^2,\omega,\omega,\omega^2,1),\ (1,\omega^2,0,0,1,\omega)\},\{(0,1,0,\omega^2,0,1),\ (1,\omega,\omega,1,\omega,\omega),\\ & (1,\omega^2,\omega,1,\omega,1),\ (1,0,\omega,\omega,\omega,1),\ (1,1,\omega,\omega,\omega,\omega),\ (0,0,0,1,0,\omega^2)\},\\ & \{(1,\omega^2,0,\omega^2,\omega^2,1),\ (1,1,\omega,\omega,0,0),\ (0,1,1,\omega,\omega^2,1),\ (1,\omega,1,\omega,\omega^2,1),\\ & (1,0,\omega^2,\omega^2,0,0),\ (0,0,0,1,1,\omega)\},\ \{(1,\omega^2,1,\omega^2,\omega^2,\omega)\ (1,0,\omega,\omega^2,0,\omega^2),\\ & (0,1,\omega,0,1,\omega^2),\ (0,1,0,0,1,0),\ (1,0,\omega^2,\omega^2,0,1),\ (1,\omega^2,0,\omega^2,\omega^2,0)\},\\ & \{(1,1,1,\omega,1,\omega^2),\ (1,\omega,\omega,0,0,1),\ (1,1,\omega,1,\omega^2,0),\ (1,\omega^2,0,0,0,1),\\ & (1,\omega^2,1,1,\omega^2,0),\ (1,\omega,0,\omega,1,\omega^2)\},\ \{(1,\omega^2,\omega,1,0,\omega),\ (1,\omega^2,\omega,\omega^2,\omega^2,\omega),\\ & (1,\omega,1,0,\omega^2,\omega^2),\ (1,\omega,1,1,1,\omega^2),\ (1,0,0,\omega^2,0,1),\ (1,0,0,0,1,1)\},\\ & \{(1,\omega^2,1,1,\omega,\omega),\ (1,\omega^2,\omega^2,\omega^2,0,1),\ (1,1,0,\omega,1,0),\ (1,1,\omega,0,\omega^2,\omega^2),\\ & (0,1,1,0,0,\omega),\ (0,1,0,1,1,0)\}\},\end{aligned}$$

$$\begin{aligned}\mathcal{O}_4 \;=\; & \{\{(1,1,\omega,\omega,1,1),\ (1,0,\omega^2,\omega^2,0,1),\ (1,\omega,1,0,\omega^2,\omega^2),\ (1,\omega^2,0,1,\omega,\omega^2),\\ & (0,1,1,0,0,\omega),\ (0,0,0,1,1,\omega)\},\ \{(1,\omega^2,1,1,\omega,\omega),\ (1,\omega,\omega,1,\omega,\omega),\\ & (1,\omega^2,0,0,0,1),\ (0,1,0,\omega^2,1,\omega),\ (1,\omega,\omega^2,0,0,1),\ (0,1,1,\omega,\omega^2,1)\},\\ & \{(1,\omega,\omega,0,0,1),\ (1,\omega^2,\omega,1,0,\omega),\ (0,1,0,0,0,0),\ (1,0,\omega,\omega^2,0,\omega^2),\\ & (1,1,\omega,\omega,0,0),\ (0,0,0,1,0,\omega^2)\},\ \{(1,\omega^2,1,\omega^2,\omega^2,\omega),(1,1,0,0,\omega,\omega),\\ & (1,\omega^2,0,\omega^2,0,1),\ (1,0,\omega,\omega,\omega,1),\ (1,1,\omega,0,\omega^2,\omega^2),\ (1,0,1,\omega,0,\omega^2)\},\\ & \{(1,\omega^2,\omega,1,\omega,1),\ (1,\omega^2,\omega,\omega,\omega^2,1),\ (1,\omega,1,0,0,1),\ (1,0,0,1,\omega,1),\\ & (1,\omega,1,\omega,\omega^2,1),\ (1,0,0,0,0,1)\},\ \{(1,\omega^2,1,1,\omega,\omega^2),\ (0,1,\omega,0,1,\omega^2),\\ & (0,0,0,1,\omega^2,0),\ (1,1,\omega,\omega,\omega,\omega),\ (1,\omega,\omega^2,0,0,0),\ (1,0,0,\omega^2,0,1)\},\\ & \{(0,1,0,0,1,0),\ (0,0,1,1,0,0),\ (1,1,\omega,\omega,1,0)\ (1,\omega^2,1,1,\omega^2,0),\\ & (1,\omega,0,0,\omega,0),\ (1,0,\omega^2,\omega^2,0,0)\},\ \{(1,1,1,\omega,1,\omega^2),\ (1,0,1,1,1,1),\\ & (0,1,0,\omega,0,\omega^2),\ (1,\omega,1,1,1,\omega^2),\ (1,\omega^2,1,\omega,1,1),\ (0,0,0,1,0,1)\},\\ & \{(0,1,0,\omega^2,0,1),\ (1,1,0,\omega,1,0),\ (1,\omega^2,0,0,1,\omega),\ (1,\omega,0,\omega,1,\omega^2),\\ & (0,0,0,1,0,0),\ (1,0,0,0,1,1)\},\ \{(1,1,1,1,0,0),\ (1,1,\omega,1,\omega^2,0),\\ & (1,\omega^2,\omega^2,\omega^2,0,0),\ (1,\omega^2,0,\omega^2,\omega^2,0),\ (0,1,\omega,1,\omega^2,0),\ (0,1,0,1,1,0)\},\\ & \{(1,\omega^2,0,\omega^2,\omega^2,1),\ (1,\omega^2,\omega^2,\omega^2,0,1),\ (0,0,1,0,\omega,1),\ (0,0,0,0,1,\omega),\\ & (1,\omega^2,\omega,\omega^2,\omega^2,\omega),\ (1,\omega^2,1,\omega^2,0,\omega)\}\}.\end{aligned}$$

The set of 66 points covered by the hyperovals in $\mathcal{O}_1$ is exactly the same subset of $\mathcal{Q}$ as that covered by the hyperovals in $\mathcal{O}_1$. Call this point set $\mathcal{Q}_0$. From the construction it follows that every hyperoval belonging to $\mathcal{O}_1$ meets every hyperoval belonging to $\mathcal{O}_4$ in at most one point. Thus each hyperoval in $\mathcal{O}_4$ meets exactly 6 hyperovals in $\mathcal{O}_1$, thereby determining 11 distinct subsets of $\mathcal{O}_1$, each of size 6. A direct MAGMA computation shows that each pair of hyperovals from $\mathcal{O}_1$ lies in exactly 3 such subsets, and hence we have a $2-(11,6,3)$ BIBD. The complementary design is a $2-(11,5,2)$ biplane, namely the Hadamard design $\mathcal{H}_{11}$.

One can also obtain the Hadamard design directly as follows. Choose some point $X \in \mathcal{Q}_0$, and take the point orbit $R$ of $X$ under a Sylow 11-subgroup of $G$. This orbit has size 11, and each hyperoval in either $\mathcal{O}_1$ or $\mathcal{O}_4$ contains exactly one point in $R$. The points in $R$ are the following:

$$\begin{aligned}R \;=\; & \{(0,1,\omega,0,1,\omega^2),\ (0,1,1,0,0,\omega),\ (1,0,0,0,0,1),\ (1,\omega,0,0,\omega,0),\\ & (1,0,1,\omega,0,\omega^2),\ (0,0,0,0,1,\omega),\ (1,1,\omega,\omega,0,0),\ (0,1,0,\omega^2,0,1),\\ & (1,\omega,1,1,1,\omega^2),\ (1,1,\omega,1,\omega^2,0),\ (0,1,0,\omega^2,1,\omega)\}.\end{aligned}$$

Thus removing the points of $R$ yields two collections of 11 disjoint ovals (actually, conics), say $\mathcal{C}_1$ and $\mathcal{C}_4$, that cover the same set of 55 points in $\mathcal{H}(5,4)$. As above, each oval in $\mathcal{C}_4$ meets 5 ovals in $\mathcal{C}_1$ in one point each, thereby determining 11 distinct subsets of $\mathcal{C}_1$ of size 5. Direct computations show that any pair of ovals from $\mathcal{C}_1$ lies in exactly 2 such subsets. Hence we again have the design $\mathcal{H}_{11}$.

# References


[1] F. Buekenhout, P. Cara, K. Vanmeerbeek, Geometries of the group PSL$(2,11)$, Geometriae Dedicata 83 (2000), 169-206.

[2] A.E. Brouwer, Unpublished paper.

[3] J. Cannon, C. Playoust, *An Introduction to MAGMA*, University of Sydney pres, Sydney, Australia, 1993.

[4] R.H. Dye, On the Conjugacy Classes of Involutions of the Unitary Groups $U_m(K)$, $SU_M(K)$, $PU_m(K)$, PSU$_m(K)$, over Perfect Fields of Characteristic 2, J. Algebra 24 (1973), 453-459.

[5] W.M. Kantor, R. Liebler, The rank 3 permutation representations of the finite classical grouops, Trans. Amer. Math. Soc. 271 (1982), 1-71.

[6] K. Metsch, L. Storme, maximal partial ovoids and maximal parial spreds in hermitian generalized quadrangles, J. Combin. Designs, submitted.

[7] R.A. Parker, Modular character tables, University of Cambridge, 1984.

[8] B. Mortimer, The modular permutation representations of the known doubly transitive groups, Proc. London Math. Soc. 41 (1980), 1-20.



A. Cossidente
Dipartimento di Matematica
Università degli Studi della Basilicata
85100 Potenza, Italy
*cossidente@unibas.it*

G. L. Ebert
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, U.S.A.
*ebert@math.udel.edu*

G. Marino
Dipartimento di Matematica e Appl.
Università di Napoli "Federico II"
80126 Napoli, Italy
*giuseppe.marino@unina2.it*