Please start each problem on a new page. Remember to justify your answers to receive full credit.

1. Suppose $Q$ is a $2m \times 2m$ unitary matrix. Let $\hat{Q}$ be the $2m \times m$ matrix consisting of the first $m$ columns of $Q$.
   (a) Write out a full (not reduced) SVD of $\hat{Q}$, in terms of $Q$ and any other matrices you wish to define.
   (b) What is $\kappa_2(\hat{Q})$?

2. Suppose $A \in \mathbb{C}^{m \times n}$ has full rank, and $A$ is diagonal:
   \[
   A = \begin{bmatrix}
   a_{11} & & & \\
   & a_{22} & & \\
   & & \ddots & \\
   0 & \cdots & & a_{nn} \\
   \vdots & & & \vdots \\
   0 & \cdots & & 0
   \end{bmatrix}
   \]
   (a) Find $A^+$, the pseudoinverse of $A$.
   (b) Find $|A^+A - I|_F$ and $|AA^+ - I|_F$, for appropriately sized identity matrices.

3. Let $D$ be a diagonal $m \times m$ matrix with positive numbers on the diagonal. Then we can define a norm for all vectors $u \in \mathbb{C}^m$ by $|u|_D = (u^* Du)^{1/2}$.
   (a) Show that $|u|_D = |v|_2$ for an appropriately defined $v$.
   (b) Suppose also that $b \in \mathbb{C}^m$, $A \in \mathbb{C}^{m \times n}$, $m \geq n$, and $A$ has full rank. Find an $x$ that minimizes $|Ax - b|_D$.

4. Find the 1-norm condition number for the problem of computing $e^{x+y}$ given the scalar values $x$ and $y$.

5. Consider the problem of finding the square root of a positive number. Suppose a computer returns exactly fl($\sqrt{\text{fl}(x)}$) in every case. Is this algorithm backward stable?

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![Image](From xkcd.com)