

1. (a) $\hat{Q} = Q \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} I$

(b) $\frac{\sigma_1}{\sigma_n} = 1$

2. (a) $A^+ = \left[\begin{array}{ccc|c} \frac{1}{a_{11}} & & & 0 \\ & \frac{1}{a_{22}} & & \\ & & \ddots & \\ & & & \frac{1}{a_{nn}} \end{array} \right]$

(b) $\|A^+A - I\|_F = \|0\|_F = 0$

$\|AA^+ - I\|_F = \left\| \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 0 \dots 0 \end{bmatrix} - I \right\|_F = \sqrt{m-n}$

3. (a) $\|u\|_D = u^* D u = (D^{1/2} u)^* (D^{1/2} u) = \|D^{1/2} u\|_2$, where
 $D^{1/2} = \text{diag}(d_{11}^{1/2}, \dots, d_{nn}^{1/2})$.

(b) $\|Ax - b\|_D = \|D^{1/2} Ax - D^{1/2} b\|_2 \Rightarrow$
 $x = (A^* (D^{1/2})^* D^{1/2} A)^{-1} A^* (D^{1/2})^* D^{1/2} b$
 $= (A^* D A)^{-1} A^* D b$

$$4. f(x, y) = e^{x+y}$$

$$J(x, y) = [e^{x+y} \quad e^{x+y}] \quad \|J\|_1 = \max\{e^{x+y}, e^{x+y}\} = e^{x+y}$$

$$K = \frac{\|J\|_1 \| [x \ y] \|_1}{\|f(x, y)\|_1} = \frac{e^{x+y} (|x| + |y|)}{e^{x+y}} = |x| + |y|$$

$$5. \tilde{f}(x) = (1+\epsilon_2) \sqrt{x(1+\epsilon_1)} = \sqrt{\tilde{x}}, \quad \text{where } \tilde{x} = x(1+\epsilon_1)(1+\epsilon_2)^2$$

$$\text{Then } \frac{\|\tilde{x} - x\|}{\|x\|} = |(1+\epsilon_1)(1+\epsilon_2)^2 - 1|$$

$$= |\epsilon_1 + 2\epsilon_2 + 2\epsilon_1\epsilon_2 + \epsilon_2^2 + \epsilon_1\epsilon_2^2|$$

$$= O(\epsilon_{\text{machine}})$$

Backward stable