Write all solutions on these sheets. Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or the computer.

1. **(25 points)** The modified Euler method is given by the tableau

\[
\begin{array}{c|cc}
0 & 1 \\
1 & 1 \\
\frac{1}{2} & \frac{1}{2} \\
\end{array}
\]

In the initial value problem \( y' = (t - y)^2, \) \( y(0) = 1, \) find \( w_1 \) if \( h = 1/3. \)

2. **(30 points)** Consider the multistep method

\[
w_{i+1} = w_i + (1 - \theta)hf_i + \theta hf_{i+1},
\]

where \( 0 \leq \theta \leq 1 \) is a constant.

   (a) Show that the method is convergent as \( h \to 0 \) for any value of \( \theta. \)

   (b) Find a value of \( \theta \) such that the order of accuracy is greater than one.

   (c) Suppose \( \theta = 1/4 \) and you wish to solve \( y' = -40y. \) Find a timestep restriction on \( h \) due to absolute stability.

3. **(a) (15 points)** Derive the BDF2 method by interpolating three values of \( w \) by a polynomial, differentiating, evaluating at a value of \( t, \) and equating to \( f_{i+1}. \)

   (b) **(10 points)** Prove that BDF2 is stable.

4. **(20 points)** The ODE \( y'' = y - y^3 \) has two stable constant solutions, \( y(t) \equiv -1 \) and \( y(t) \equiv 1. \) (That is, perturbations to these solutions do not grow with time.) Which IVP method would give stable long-time approximations to these solutions using \( h = 1/20, \) Euler or Midpoint? Explain your answer carefully.