NAME

MATH341, Spring 2007
Exam 2: April 6

Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. You must justify answers to receive full credit. You may not use calculators or notes.

\[ mx'' + cx' + kx = F_0 \cos \omega t \] has solution \( x = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t + C \cos (\omega t - \alpha) \),

where

\[ C = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}, \quad \tan \alpha = \frac{c\omega}{k - m\omega^2}, \quad 0 < \alpha < \pi, \]

unless \( c = 0 \) and \( \omega = \omega_0 \).

1. (10 points) Determine whether the functions \( f(x) = 1, g(x) = \cos^2 x, \) and \( h(x) = \cos 2x \) are linearly dependent or independent on the real line.

2. (20 points) Solve the initial-value problem \( y''' - 2y'' + y' = 0, \) \( y(0) = 1, y'(0) = 1, y''(0) = 2. \)

3. (20 points) Write down the form for a particular solution \( y_p \), but do not try to find the coefficients, for the equation \( y''' - 2y'' + y' = 5 + 3e^x + e^{-x} - e^x \cos 3x. \) (This has the same left-hand side as problem #2.)

4. (20 points) Find the general solution of \( y'' + 2y' + 2y = 10 \sin x. \)

5. (20 points) A mass of 2 kg is attached to a spring and allowed to hang freely, causing a static displacement of 0.05 \( g \) m, where \( g \) is gravitational acceleration. Assume a damping coefficient \( c = 16 \) kg/s.

   (a) Is this oscillator overdamped, critically damped, or underdamped? Explain.

   (b) Suppose the system is subjected to a driving force \( F_0 \cos \omega t. \) Find the practical resonant frequency \( \omega, \) or show that none exists.

6. (10 points) Write \( xy''' - yy' = e^x, \) where \( y = y(x), \) as an equivalent first-order system of differential equations.