

MATH 241, Spring 2009  
Exam 3: May 13

NAME \_\_\_\_\_ Tue/Thurs discussion time \_\_\_\_\_

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (10 points) Find all inflection points and the intervals of concavity in  $0 \leq \theta \leq \pi$  for  $f(\theta) = \cos(\theta) + \frac{1}{4}\theta^2$ .

$$f'(\theta) = -\sin \theta + \frac{1}{2}\theta$$

$$f''(\theta) = 0 \Rightarrow \cos \theta = \frac{1}{2}$$

$$f''(\theta) = -\cos \theta + \frac{1}{2}$$

$$\theta = \frac{\pi}{3} \text{ in } [0, \pi]$$

inflection point.

$\cos \theta > \frac{1}{2}$  for  $0 \leq \theta < \frac{\pi}{3}$ , so  $f'' < 0$  concave down

$\cos \theta < \frac{1}{2}$  for  $\frac{\pi}{3} < \theta \leq \pi$ , so  $f'' > 0$  concave up

2. (15 points) Find the minimum and maximum values of  $f(t) = t\sqrt{18-t^2}$  on the interval  $[0,4]$ .

$$f'(t) = \sqrt{18-t^2} + \frac{1}{2}t(18-t^2)^{-1/2}(-2t)$$

$$\text{critical number : } 0 = (18-t^2)^{1/2} - t^2(18-t^2)^{-1/2}$$

$$0 = (18-t^2)^1 - t^2 = 18 - 2t^2$$

$$t = 3 \text{ or } t = -3$$

(not in  $[0,4]$ )

$$f(0) = 0 \quad \leftarrow \text{min value}$$

$$f(3) = 3\sqrt{9} = 9 \quad \leftarrow \text{max value}$$

$$f(4) = 4\sqrt{2} < 9$$

3. (10 points) Find all local minimum and local maximum points of  $g(x) = 100 - 18x^2 + x^4$ .

$$g'(x) = -36x + 4x^3 = 4x(x^2 - 9) = 4x(x-3)(x+3)$$

critical numbers  $x=0, x=3, x=-3$

$$g''(x) = -36 + 12x^2$$

$$g''(0) = -36 < 0 \quad \text{local max}$$

$$g''(3) = -36 + 12 \cdot 9 > 0 \quad \text{local min}$$

$$g''(-3) = -36 + 12 \cdot 9 > 0 \quad \text{local min}$$

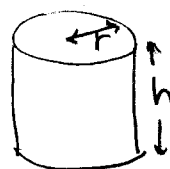
OR

	$4x$	$x-3$	$x+3$	$g'(x)$	
$x < -3$	-	-	-	-	
$-3 < x < 0$	-	-	+	+	← min
$0 < x < 3$	+	-	+	-	← max
$x > 3$	+	+	+	+	← min

4. (15 points) A cylindrical metal can with no lid is supposed to hold  $1000\pi$  cm<sup>3</sup> of liquid. Find the dimensions of the can that minimizes the amount of material used.

$$1000\pi = V = \pi r^2 h \quad (\text{constraint})$$

Material used  $\rightarrow$  surface area



$$S = 2\pi r h + \pi r^2 \quad (\text{minimize})$$

(sides)      (bottom)

$$S = 2\pi r \left( \frac{1000\pi}{\pi r^2} \right) + \pi r^2 = \frac{2000\pi}{r} + \pi r^2$$

$$S'(r) = -\frac{2000\pi}{r^2} + 2\pi r$$

$$0 = S'(r) \Rightarrow 2\pi r = \frac{2000\pi}{r^2}$$

$$r^3 = 1000$$

$$r = 10 \text{ cm}$$

$$h = \frac{1000}{r^2} = \frac{1000}{100} = 10 \text{ cm}$$

5. (10 points) A car traveling 99 ft/sec begins to experience deceleration of  $e^t$  starting at  $t = 0$ . How far will it have traveled between  $t = 0$  and  $t = 1$ ? (No need to simplify the number.)

$$v(t) = \int a(t) dt = \int -e^t dt = -e^t + C$$

$$99 = v(0) = -1 + C, \quad \text{so } C = 100$$

$$s(t) = \int v(t) dt = -e^t + 100t + B$$

$$\begin{aligned} s(1) - s(0) &= (-e^1 + 100 + B) - (-e^0 + 0 + B) \\ &= -e + 101 \text{ ft.} \end{aligned}$$

6. (10 points) Write down the Riemann sum  $R_5$  that approximates  $\int_{-5}^{10} \cos(x) dx$ , using right endpoints of five intervals. Do not try to simplify or evaluate the number.

$$\Delta x = \frac{10 - (-5)}{5} = 3$$

$$\begin{array}{lll} x_0 = -5 & x_2 = 1 & x_4 = 7 \\ x_1 = -2 & x_3 = 4 & x_5 = 10 \end{array}$$

$$R_5 = \sum_{i=1}^5 f(x_i) \Delta x = 3 \left( \cos(-2) + \cos(1) + \cos(4) + \cos(7) + \cos(10) \right)$$

7. (10 points) Find  $\frac{d}{dx} \left[ \int_1^{1/x} \sinh^3(s) ds \right]$ .

$$u = \frac{1}{x} \quad \left( \frac{d}{du} \int_1^u \sinh^3(s) ds \right) \left( \frac{du}{dx} \right)$$

$$= \sinh^3(u) \cdot \left( -\frac{1}{x^2} \right)$$

$$= -\frac{\sinh^3(1/x)}{x^2}$$

8. (10 points) Evaluate  $\int_0^{\pi/6} 2 \cos(\theta) d\theta$ .

$$[2 \sin \theta]_0^{\pi/6} = (2 \sin \frac{\pi}{6} - 2 \sin 0) = 1$$

9. (10 points) Evaluate  $\int x(\sqrt{x}-1) dx$ .

$$\int (x^{3/2} - x) dx = \frac{2}{5} x^{5/2} - \frac{1}{2} x^2 + C$$