

MATH 241, Spring 2009  
Exam 1: March 11

NAME \_\_\_\_\_ Discussion section time \_\_\_\_\_

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

For this exam, you must use a limit formula to compute any derivative.

1. (6 points each) Let  $f(x) = e^{2x} + 1$ .

(a) Find a formula for  $f^{-1}(x)$ . (b) Find the domain of  $f$  and the domain of  $f^{-1}$ .

$$(a) \quad y = e^{2x} + 1$$

$$y - 1 = e^{2x}$$

$$x = \frac{1}{2} \ln(y - 1) = f^{-1}(y)$$

$$f^{-1}(x) = \frac{1}{2} \ln(x - 1)$$

$$(b) \quad e^{2x} \text{ defined for all } x$$

$$\text{domain of } f = (-\infty, \infty)$$

$$\ln(x - 1) \text{ defined only if } x - 1 > 0$$

$$\text{domain of } f^{-1} = (1, \infty)$$

2. (12 points) Solve  $\ln(x) - \ln(x - 1) = \ln(2)$  for  $x$ .

$$\ln\left(\frac{x}{x-1}\right) = \ln(2)$$

$$\frac{x}{x-1} = 2$$

$$x = 2x - 2$$

$$x = 2$$

3. (12 points) Find the exact value of  $\arccos\left[\cos\left(\frac{5\pi}{4}\right)\right]$ .

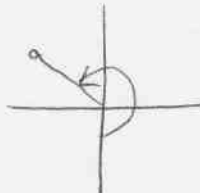
$$\cos\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}}$$



$$\cos(\theta) = \frac{1}{\sqrt{2}}$$

and

$$0 \leq \theta \leq \pi$$



$$\theta = \frac{3\pi}{4}$$

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4. (4 points each) Let  $f(x) = \begin{cases} \sqrt{|1+x|} & \text{if } x < 0, \\ 1-x & \text{if } x \geq 0. \end{cases}$

Evaluate each limit, or write DNE if it does not exist.

(a)  $\lim_{x \rightarrow 0^-} f(x)$

(b)  $\lim_{x \rightarrow 0^+} f(x)$

(c)  $\lim_{x \rightarrow 0} f(x)$

(a)  $\lim_{x \rightarrow 0^-} \sqrt{|1+x|} = \lim_{x \rightarrow 0^-} \sqrt{1+x} = 1$

(b)  $\lim_{x \rightarrow 0^+} 1-x = 1$

(c)  $\lim_{x \rightarrow 0} f = \lim_{x \rightarrow 0^-} f = \lim_{x \rightarrow 0^+} f = 1$

Reminder: For this exam, you must use a limit formula to compute any derivative.

5. (12 points) Find the limit, or write DNE if it does not exist.

$$\lim_{x \rightarrow 1^-} \frac{x^2 - 16}{x^2 - 5x + 4}$$

$$\lim_{x \rightarrow 1^-} \frac{(x+4)(\cancel{x-4})}{(\cancel{x-4})(x-1)} = \frac{4^-}{0^-} = -\infty$$

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6. (8 points each) Find each limit, or write DNE if it does not exist.

$$(a) \lim_{x \rightarrow \infty} \frac{1}{\ln(x)}$$

$$(b) \lim_{x \rightarrow \infty} \frac{4x^2 - 16}{x^4 + 1}$$

$$(a) \ln(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty$$

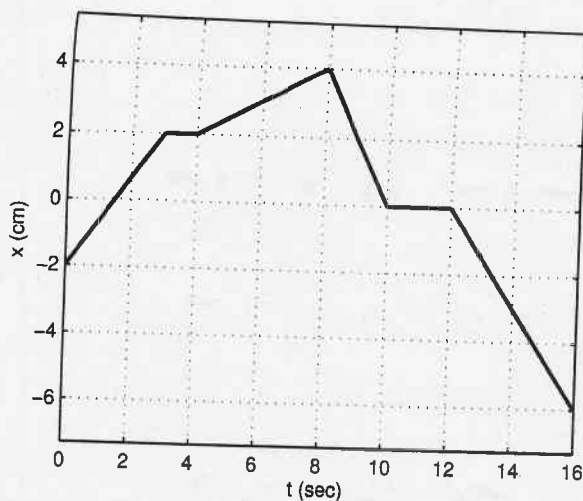
$$\text{Thus } \frac{1}{\ln(x)} \rightarrow 0 \text{ as } x \rightarrow \infty$$

$$(b) \lim_{x \rightarrow \infty} \frac{4x^2 - 16}{x^4 + 1} = \frac{x^{-4}}{x^{-4}}$$

$$= \lim_{x \rightarrow \infty} \frac{4x^{-2} - 16x^{-4}}{1 + x^{-4}} = \frac{0}{1} = 0$$

Reminder: For this exam, you must use a limit formula to compute any derivative.

7. (4 points each) A particle moves horizontally in a straight line according to the position function  $x(t)$ , whose graph is shown here.



- (a) What is the average velocity over  $0 \leq t \leq 16$ ?  
(b) At what time(s), if any, is the particle moving to the right?  
(c) At what time(s), if any, is the instantaneous velocity undefined?

$$(a) \quad \frac{\Delta x}{\Delta t} = \frac{-6 - (-2)}{16 - 0} = -\frac{1}{4}$$

$$(b) \quad \text{slope} > 0 \quad \text{for} \quad (0, 3), (4, 8)$$

(c) corners indicate no differentiability  
at  $t = 3, 4, 8, 10, 12,$

8. (12 points) Find the equation of the line tangent to  $y = 3x^2$  at the point  $(1,3)$ .

$$f(x) = 3x^2, \text{ find } f'(1) = \lim_{h \rightarrow 0} \frac{3(1+h)^2 - 3(1)^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(1+2h+h^2) - 3}{h} = \lim_{h \rightarrow 0} \frac{6h - 3h^2}{h}$$

$$= 6$$

$$(y-3) = 6(x-1)$$

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