MATH 241, Fall 2008
Exam 3: December 3

NAME _____ **Discussion section** _____

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (15 points) Find all the local minima and local maxima of $x^4 - 8x^2 + 8$.

$f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$

critical numbers are $x = 0$, $x = -2$, $x = 2$

$f''(x) = 12x^2 - 16$

$f''(0) = -16$ $x = 0$ is local max

$f''(-2) = 32$ $x = -2$ is local min

$f''(2) = 32$ $x = 2$ is local min

2. (15 points) Find the absolute minimum and absolute maximum values of $f(x) = \dfrac{x}{x^2+4}$ on the interval $[0,4]$.

$$f'(x) = \frac{(x^2+4) - x(2x)}{(x^2+4)^2} = \frac{4-x^2}{(4+x^2)^2}$$

Critical numbers ~~$x=-2$~~, $x=2$.

~~$x=-2$~~ not in $[0,4]$

$f(0) = 0$ ← abs min value

$f(2) = \frac{2}{8} = \frac{1}{4}$ ← abs. max value

$f(4) = \frac{4}{20} = \frac{1}{5}$

3. (10 points) Evaluate $\frac{d}{dx}\left[\int_1^{\cosh(x)} t\sin(t)\,dt\right]$. (No need to simplify the result.)

(FTC1) $u = \cosh(x)$, $\frac{d}{dx}[\quad] = \frac{d}{du}[\quad]\frac{du}{dx}$

$= u\sin(u)\sinh(x) = \cosh(x)\sin(\cosh(x))\sinh(x)$

4. (10 points) Find where the graph of $y = x + \cos 2x$ is concave up for $0 \le x \le \pi$.

$y' = 1 - 2\sin 2x$

$y'' = -4\cos 2x$

Concave up when

$-4\cos 2x > 0$

$\cos 2x < 0$

$\frac{\pi}{4} < x < \frac{3\pi}{4}$

5. (10 points) Evaluate $\int_{-1}^{1}(3t-6)(t-2)\,dt$.

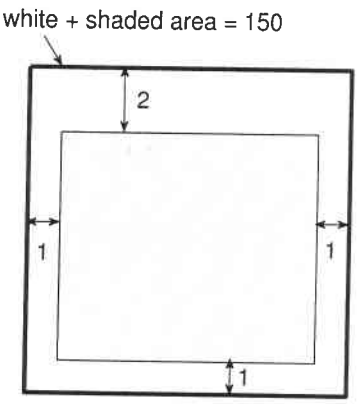
$$\int_{-1}^{1}(3t^2-12t+12)\,dt = \left[t^3-6t^2+12t\right]_{-1}^{1}$$

$$= (1-6+12)-(-1-6-12)$$

$$= 26$$

6. (10 points) Evaluate $\int_{1}^{3}\frac{9}{u^4}\,du$.

$$\int_{1}^{3}9u^{-4}\,du = \left[-3u^{-3}\right]_{1}^{3} = \left(-\frac{1}{9}\right)-(-3) = \frac{26}{9}$$



7. (15 points) A poster is to have a total area of 150 in^2. When printed, it must leave a top margin of 2 in, and margins on the sides and bottom of 1 in each. What are the overall poster dimensions that give the maximum possible printed area? (The diagram does not show the solution!)

x = width, y = height

maximize $A = (x-2)(y-3)$ $\quad (x>2,\ y>3)$

constrained by $xy = 150$ (given)

$\therefore\ A(x) = (x-2)\left(\frac{150}{x} - 3\right) = 150 - 3x - \frac{300}{x} + 6$

$A' = -3 + \frac{300}{x^2}$ $\quad$ critical number $x^2 = 100$

$x = 10$

A' changes sign from positive to negative at $x = 10$, so this is an absolute max.

$x = 10,\ y = 15$

8. (15 points) Suppose the height h of a building 120 meters away is calculated by measuring the angle θ it subtends, so that $h = 120\tan(\theta)$. If $\theta = \pi/6$ with a 5% error, estimate the absolute error in the height.

$dh = 120 \sec^2\theta \, d\theta$ $\qquad \frac{d\theta}{\theta} = 5\% = \frac{1}{20}$

$= 120 \left(\cos\frac{\pi}{6}\right)^{-2} \frac{1}{20} \cdot \frac{\pi}{6}$

$= \frac{\pi}{(\sqrt{3}/2)^2} = \frac{4\pi}{3}$ meters