

MATH 241, Fall 2008
Exam 2: November 3

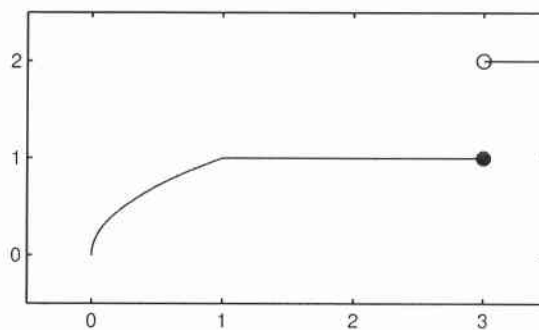
NAME _____

Discussion section _____

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. Unless otherwise noted, you must justify all answers to receive full credit. You may not use calculators, notes, or any other kinds of aids.

1. (12 points) At which value(s) of x is this function not differentiable? Give short reasons for each value you state.



$x = 0$ — vertical tangent

$x = 1$ — corner

$x = 3$ — discontinuous

2. (12 points) Find y' if $y = \frac{2^x}{1-x^2}$.

$$y' = \frac{(1-x^2)(\ln 2)2^x - 2^x(-2x)}{(1-x^2)^2} = 2^x \frac{\ln(2)(1-x^2) + 2x}{(1-x^2)^2}$$

3. (12 points) Find y'' if $y = x^5 + \frac{2}{e^{2x}}$.

$$y = x^5 + 2e^{-2x}$$

$$y' = 5x^4 - 4e^{-2x}$$

$$y'' = 20x^3 + 8e^{-2x}$$

4. (12 points) Find the line tangent to the hyperbola $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

$$2x + 2y + 2xy' - 2yy' + 1 = 0$$

$$y' = \frac{-2x - 2y - 1}{2x - 2y} \quad @ \quad x=1, y=2 : \quad y' = \frac{-2-4-1}{2-4} = \frac{7}{2}$$

$$y - 2 = \frac{7}{2}(x - 1)$$

5. (12 points) Find $\frac{d}{dx}[(\cos x)^x]$.

$$y = (\cos x)^x \Rightarrow \ln y = x \ln(\cos x)$$

$$\Rightarrow \frac{y'}{y} = \ln(\cos x) + x \frac{-\sin x}{\cos x}$$

$$\Rightarrow y' = (\cos x)^x [\ln(\cos x) - x \tan x]$$

6. (12 points) At what point does the curve $y = [\ln(x+3)]^2$ have a horizontal tangent?

$$y' = 2 [\ln(x+3)] \frac{1}{x+3}$$

If $y' = 0$, then $\ln(x+3) = 0$

$$x+3 = e^0 = 1$$

$$x = -2 \Rightarrow y = [\ln(1)]^2 = 0$$

@ (-2, 0)

7. (14 points) A cup of tea sits in a room kept at 20°C and cools from 80°C to 60°C in half an hour. What will be the temperature of the tea after another half hour? Simplify your answer for full credit.

$$\left. \begin{aligned} y(0) &= 80 - 20 = 60 \\ y(30) &= 60 - 20 = 40 \end{aligned} \right\} \text{ given} \quad \text{Find } y(60) + 20.$$

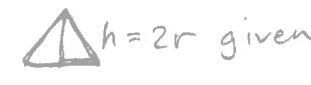
$$y(t) = y(0) e^{kt} = 60 e^{kt}$$

$$40 = 60 e^{30k} \Rightarrow k = \frac{1}{30} \ln\left(\frac{2}{3}\right) \quad (\text{per min})$$

$$y(60) = 60 e^{2 \ln(2/3)} = 60 e^{\ln(4/9)} = 240/9$$

$$T(60) = 20 + \frac{240}{9}$$

8. (14 points) Gravel is dumped onto a conical pile at a rate of 36 ft^3 per second. The gravel always shifts so that the base diameter of the pile equals its height. At what rate is the height of the pile increasing when it is 6 ft high? (The volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.)



$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{36}{\pi/4 \cdot 6^2} = \frac{4}{\pi} \text{ ft/sec}$$