

MATH 241, Fall 2008
Exam 1: October 3

NAME SOLUTIONS

Discussion section _____

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (10 points) If $f(x) = 2e^{-x}$, find a formula for $f^{-1}(x)$.

$$y = 2e^{-x}$$

$$\frac{y}{2} = e^{-x}$$

$$\ln\left(\frac{1}{2}y\right) = -x$$

$$x = -\ln\left(\frac{1}{2}y\right)$$

swap

$$y = -\ln\left(\frac{1}{2}x\right)$$

$$= \ln\left(\frac{2}{x}\right) \text{ or } \ln(2) - \ln(x)$$

$$f^{-1}(x) = \ln\left(\frac{2}{x}\right)$$

2. (5 points each) State the domain and range of each function.

(a) $f(x) = \tan^{-1}(\sqrt{x})$

(b) $g(x) = \ln(2^x - 1)$

\sqrt{x} maps $[0, \infty)$ to $[0, \infty)$

\tan^{-1} maps $[0, \infty)$ to $[0, \frac{\pi}{2})$



domain = $[0, \infty)$

range = $[0, \frac{\pi}{2})$

\ln requires that

$$2^x - 1 > 0$$

$$2^x > 1$$

$$x \ln(2) > \ln(1) = 0$$

$$x > 0$$

2^x maps $(0, \infty)$ to $(1, \infty)$

$2^x - 1$ maps $(0, \infty)$ to $(0, \infty)$

\ln maps $(0, \infty)$ to $(-\infty, \infty)$

domain = $(0, \infty)$

range = $(-\infty, \infty)$

3. (6 points each) Simplify each expression into a simple number.

(a) $\log_2(6) - \log_2(15) + \log_2(10)$

$$= \log_2\left(\frac{6}{15}\right) + \log_2(10)$$

$$= \log_2\left(\frac{60}{15}\right)$$

$$= \log_2(4) = \boxed{2}$$

(b) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

↓

$$\cos \theta = \frac{1}{2} \text{ and}$$

$$0 \leq \theta \leq \pi$$

↓

$$\theta = \boxed{\frac{\pi}{3}}$$

4. (12 points) Find $\lim_{t \rightarrow 2} \frac{t^2 - 2t}{|2 - t|}$, or show it does not exist.

$$\frac{t^2 - 2t}{|2 - t|} = \begin{cases} \frac{t^2 - 2t}{2 - t} & \text{if } 2 - t > 0, \text{ or } t < 2 \\ \frac{t^2 - 2t}{-(2 - t)} & \text{if } 2 - t \leq 0, \text{ or } t \geq 2 \end{cases}$$

$$= \begin{cases} \frac{t(t-2)}{(-1)(t-2)} & \text{if } t < 2 \\ \frac{t(t-2)}{t-2} & \text{if } t \geq 2 \end{cases}$$

So $\lim_{t \rightarrow 2^-} f(t) = \frac{2}{(-1)} = -2$, $\lim_{t \rightarrow 2^+} f(t) = 2$

different, so DNE

5. (6 points each) Find all points where the given function is discontinuous. If the function is continuous everywhere, write NONE. (Remember to justify your answers.)

$$(a) f(x) = \frac{\sin(x)}{x}$$

$$(b) g(x) = \begin{cases} 1, & \text{if } x < 0, \\ \cos(x), & \text{if } 0 \leq x < \pi, \\ \sin(x), & \text{if } x \geq \pi. \end{cases}$$

$f(0)$ not defined, so

f not continuous at $x=0$

$$\lim_{x \rightarrow 0^-} g(x) = 1 = \lim_{x \rightarrow 0^+} g(x)$$

$$\text{and } g(0) = 1$$

$$\lim_{x \rightarrow \pi^-} g(x) = -1$$

$$0 = \lim_{x \rightarrow \pi^+} g(x)$$

$$\lim_{x \rightarrow \pi} g(x) \text{ DNE}$$

$$x = \pi$$

6. (15 points) Find all of the vertical and horizontal asymptotes to the graph of $y = \frac{x^3 - 3}{2x(x^2 - 1)}$.
It is not necessary to draw the graph.

Vertical: denom = 0 $2x(x+1)(x-1) = 0$

$$\boxed{x=0, x=-1, x=1}$$

horizontal: $\lim_{x \rightarrow \infty} \frac{x^3 - 3}{2x^3 - 2x} \cdot \frac{x^{-3}}{x^{-3}}$

$$= \lim_{x \rightarrow \infty} \frac{1 - 3x^{-3}}{2 - 2x^{-2}} = \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}}$$

(same as $x \rightarrow -\infty$)

7. (15 points) Using one of our limit formulas for the derivative, find the line tangent to $y = 3 + 4x^2$ at the point $(-1, 7)$.

$$f(x) = 3 + 4x^2 \quad \text{find } f'(-1)$$

$$f'(-1) = \lim_{x \rightarrow -1} \frac{3 + 4x^2 - 7}{x + 1} = \lim_{x \rightarrow -1} \frac{4(x^2 - 1)}{x + 1}$$

$$= \lim_{x \rightarrow -1} 4(x - 1) = -8$$

$$\boxed{(y - 7) = -8(x + 1)}$$

$$\text{or } y = -8x - 1$$

8. (14 points) A falling particle has height described by the equation $s = 1/t$. Find its velocity when $t = 3$.

$$s'(3) = \lim_{t \rightarrow 3} \frac{\frac{1}{t} - \frac{1}{3}}{t-3} \cdot \frac{3t}{3t}$$

$$= \lim_{t \rightarrow 3} \frac{3-t}{3t(t-3)}$$

$$= \lim_{t \rightarrow 3} \frac{-1}{3t} = \boxed{-\frac{1}{9}} \quad (\text{units not given})$$