1. a. Improper Integral of Type I because of an infinite interval of integration.
   b. Improper Integral of Type II because sec(x) has an infinite discontinuity at \( x = \frac{\pi}{2} \).
   c. Improper Integral of Type II because of the infinite discontinuity at \( x = 2 \).
   d. Improper Integral of Type I because of an infinite interval of integration.

11. \[ \int_{-\infty}^{\infty} \frac{x}{(1 + x^2)} \, dx = \int_{-\infty}^{0} \frac{x}{(1 + x^2)} \, dx + \int_{0}^{\infty} \frac{x}{(1 + x^2)} \, dx. \]
    Now consider
    \[ \int_{-\infty}^{0} \frac{x}{(1 + x^2)} \, dx = \lim_{t \to -\infty} \int_{t}^{0} \frac{x}{(1 + x^2)} \, dx = \lim_{t \to -\infty} \left[ \frac{1}{2} (1 + x^2) \right]_{t}^{0} = \lim_{t \to -\infty} \left( 0 + \frac{1}{2} \ln(1 + t^2) \right) = -\infty \]
    Since this integral diverges, then \( \int_{-\infty}^{\infty} \frac{x}{(1 + x^2)} \, dx \) also diverges.

22. \[ \int_{-\infty}^{\infty} e^{x} \, dx = \lim_{t \to -\infty} \int_{t}^{0} e^{x} \, dx + \lim_{t \to 0} \int_{0}^{t} e^{-x} \, dx = \lim_{t \to -\infty} \left[ e^{x} \right]_{t}^{0} + \lim_{t \to 0} \left[ -e^{-x} \right]_{0}^{t} = 1 + 1 = 2 \] so the integral is convergent.

28. \[ \int_{0}^{3} \frac{1}{x \sqrt{x}} \, dx = \lim_{t \to 0^{+}} \int_{t}^{3} \frac{1}{\sqrt{x}} \, dx = \lim_{t \to 0^{+}} \left[ \frac{2}{\sqrt{x}} \right]_{t}^{3} = \lim_{t \to 0^{+}} \left( \frac{-2}{\sqrt{3}} + \frac{2}{\sqrt{t}} \right) = \infty, \text{ divergent}. \]

33. \[ \int_{0}^{33} (x - 1)^{-1/5} \, dx = \lim_{t \to 0^{+}} \int_{0}^{t} (x - 1)^{-1/5} \, dx + \lim_{t \to 33^{+}} \int_{t}^{33} (x - 1)^{-1/5} \, dx \]
    \[ = \lim_{t \to 0^{+}} \left[ \frac{5}{4} (x - 1)^{4/5} \right]_{0}^{1} + \lim_{t \to 33^{+}} \left[ \frac{5}{4} (x - 1)^{4/5} \right]_{t}^{33} = -\frac{5}{4} + 20 = \frac{75}{4} \] so the integral is convergent.

50. For \( x \geq 1, \frac{2 + e^{-x}}{x} > \frac{x}{x} = 1 \) (This is because \( e^{-x} > 0 \)). Now,
    \[ \int_{1}^{\infty} \frac{1}{x} \, dx \] is divergent by Equation 2 in section 8.8 since the exponent of \( x \) is equal to 1.
    Thus, by the comparison theorem, the original integral is divergent.
51. For \( x \geq 1 \) we have that \( x + e^{2x} > e^{2x} > 0 \) so \( \frac{1}{x + e^{2x}} \leq \frac{1}{e^{2x}} = e^{-2x} \). Now looking at this integral:

\[
\int_{1}^{\infty} e^{-2x} \, dx = \lim_{t \to \infty} \int_{1}^{t} e^{-2x} \, dx = \lim_{t \to \infty} \left[ \frac{-1}{2} e^{-2x} \right]_{1}^{t} = \lim_{t \to \infty} \left[ \frac{-1}{2} e^{-2t} + \frac{1}{2} e^{-2} \right] = \frac{1}{2} e^{-2}
\]

Thus the original integral is convergent by the comparison theorem.
6. a) \( x = 1 + t, \ y = 5 - 2t, \ -2 \leq t \leq 3 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>( y )</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

b) Since \( x = 1 + t \), we have \( t = x - 1 \), so we can plug this back into the equation for \( y \) and we have \( y = 5 - 2(x - 1) = 7 - 2x \). And we have, 
\(-2 \leq t \leq 3 \Rightarrow -2 \leq x - 1 \leq 3 \Rightarrow -1 \leq x \leq 4 \).

8. a) \( x = 1 + 3t, \ y = 2 - t^2 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>-8</td>
<td>-5</td>
<td>-2</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>( y )</td>
<td>-7</td>
<td>-2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>-2</td>
<td>-7</td>
</tr>
</tbody>
</table>

b) Since \( x = 1 + 3t \), we have \( t = \frac{x - 1}{3} \), and therefore \( y = 2 - \frac{(x - 1)^2}{9} \).
11.a.) \( x = \sin \theta, \ y = \cos \theta, \ 0 \leq \theta \leq \pi \), therefore \( x^2 + y^2 = \sin^2 \theta + \cos^2 \theta = 1 \), and since \( 0 \leq \theta \leq \pi \) we have \( \sin \theta \geq 0 \Rightarrow x \geq 0 \).

b.)

12.a.) \( x = 4 \cos \theta, \ y = 5 \sin \theta, \ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \), therefore we have

\[
\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \cos^2 \theta + \sin^2 \theta = 1,
\]

which is an ellipse with x-intercepts \((\pm 4, 0)\) and y-intercepts \((0, \pm 5)\) with \( x \geq 0 \).

b.)

16.a.) \( x = \ln(t), \ y = \sqrt{t}, \ t \geq 1 \), so we have \( x = \ln(t) \Rightarrow e^x = t \), and hence \( y = \sqrt{e^x} = e^{x/2} \) and \( x \geq 0 \).

b.)
24. a.) From the first graph we have \(1 \leq x \leq 2\) and from the second graph we have \(-1 \leq y \leq 1\). The only choice that satisfies either of those conditions is III.

b.) From the first graph, the values of \(x\) cycle through the values from -2 to 2 four times. From the second graph, the values of \(y\) cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

c.) From the first graph, the values of \(x\) cycle through the values from -2 to 2 three times. From the second graph we have \(0 \leq y \leq 2\). Choice IV satisfies these conditions.

d.) From the first graph, the values of \(x\) cycle through the values from -2 to 2 two times. From the second graph, the values of \(y\) do the same thing. Choice II satisfies these conditions.