7.7: 12, 15, 17, 24, 44, 49, 58, 80

12. \( \lim_{x \to 0} \frac{e^{3x} - 1}{x} = \frac{0}{0} \) so use L’hopital’s Rule to get: \( \lim_{x \to 0} \frac{e^{3x} - 1}{x} = \lim_{x \to 0} \frac{3e^{3x}}{1} = 3 \)

15. \( \lim_{x \to 0} \frac{\ln x}{x} = \frac{\infty}{\infty} \) so use L’hopital’s Rule to get: \( \lim_{x \to 0} \frac{\ln x}{x} = \lim_{x \to 0} \frac{1/x}{1} = 0 \)

17. \( \lim_{x \to 0^+} \frac{\ln x}{x} = -\infty \) since dividing by a small \( x \) makes the quotient bigger, L’hopital’s rule doesn’t apply.

24. \( \lim_{x \to 0} \frac{\sin x}{\sinh x} = \frac{0}{0} \) so use L’hopital’s Rule to get: \( \lim_{x \to 0} \frac{\sin x}{\sinh x} = \lim_{x \to 0} \frac{\cos x}{\cosh x} = 1 \)

44. \( \lim_{x \to \infty} x \tan(1/x) = \infty(\infty) \) so use L’hopitals Rule to get

\[
\lim_{x \to \infty} \frac{\tan x}{1/x} = \lim_{x \to \infty} \frac{\sec^2(1/x)(-1/x^2)}{(-1/x^2)} = \lim_{x \to \infty} \sec^2 x = 1
\]

49. \( \lim_{x \to \infty} x - \ln x = \infty - \infty \) so factor out an \( x \) to use L’Hopital’s Rule: \( \lim_{x \to \infty} x(1 - \frac{\ln x}{x}) \) looking just at the particular limit \( \lim_{x \to \infty} \frac{\ln x}{x} = \lim_{x \to \infty} \frac{1/x}{1} = 0 \) then \( \lim_{x \to \infty} x(1 - \frac{\ln x}{x}) = \infty \).

58. Let \( y = \lim(e^x + x)^{1/x} \), Taking the natural log of both sides gives:

\( \ln y = \lim_{x \to \infty} \frac{\ln(e^x + x)}{x} \) using L’Hopital’s Rule: \( \ln y = \lim_{x \to \infty} \frac{e^x + 1}{e^x + x} = \lim_{x \to \infty} \frac{e^x}{e^x} = \lim_{x \to \infty} e^x = 1 \)

Now since \( \ln y = 1 \) then this means \( y = e^1 = e \) so \( \lim(e^x + x)^{1/x} = e \).

80.

a) \( \lim_{t \to \infty} v = \frac{mg}{c} \lim_{t \to \infty} \left(1 - e^{-ct/m}\right) = \frac{mg}{c} \)

b) \( \lim_{m \to \infty} \frac{g}{c} \lim_{m \to \infty} \left(1 - e^{-ct/m}\right) = \frac{g}{c} (ct) \lim_{m \to \infty} \left(\frac{e^{-ct/m}(-1/m^2)}{1/m^2}\right) = gt \lim_{m \to \infty} (e^{-ct/m}) = gt \)

So the velocity of a heavy falling object is approximately proportional to the time \( t \).
8.1: 2, 9, 18, 22, 42

2. Let \( u = \theta, dv = \sec^2 \theta \) then \( du = d\theta, v = \tan \theta \)
   
   so \( \int \theta \sec^2 \theta = \theta \tan \theta - \int \tan \theta d\theta = \theta \tan \theta - \ln |\sec \theta| + C \)

9. Let \( u = \ln(2x + 1), dv = dx \) then \( du = \frac{2}{2x + 1}, v = x \)

   so \( \int \ln(2x + 1)dx = \ln(2x + 1)x - \int \frac{2x}{2x + 1}dx \)

   Now, \( \int \frac{2x}{2x + 1}dx = \int \frac{(2x + 1) - 1}{(2x + 1)}dx = \int 1 - \frac{1}{2x + 1}dx = x - \frac{1}{2} \ln |2x + 1| + C \)

   So \( \int \ln(2x + 1)dx = \ln(2x + 1)x - x + \frac{1}{2} \ln |2x + 1| + C \)

18. Let \( u = y, dv = \cosh(ay)dy \) then \( du = dy, v = \frac{\sinh(ay)}{a} \)

   so \( \int y \cosh(ay)dy = \frac{y \sinh(ay)}{a} - \frac{1}{a} \int \sinh(ay)dy = \frac{y \sinh(ay)}{a} - \frac{1}{a^2} \cosh(ay) + C \)

22. Let \( u = \ln t, dv = \sqrt{t}dt \) then \( du = \frac{1}{t} dt, v = \frac{2}{3} t^{3/2} \)

   so \( \int \sqrt{t} \ln t dt = \left[ \frac{2}{3} t^{3/2} \ln t \right]_1^4 - \frac{2}{3} \int t^{1/2} dt = \left[ \frac{2}{3} t^{3/2} \ln t \right]_1^4 - \frac{4}{9} \left[ t^{3/2} \right]_1^4 = \frac{16}{3} \ln 4 - \frac{28}{9} \)

42.

a) \( u = \cos^{n-1}, dv = \cos xdx \) then \( du = -(n-1)(\cos^{n-2} x)(\sin x), v = \sin x \) so

\[
\int \cos^n dx = \cos^{n-1} x \sin x + \int \sin x (n-1) \cos^{n-2} xdx
\]

\[
= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x(1-\cos^2 x)dx
\]

\[
= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} xdx - (n-1) \int \cos^n xdx
\]

Now, adding \((n-1)\int \cos^n xdx\) to both sides gives

\[
n \int \cos^n xdx = \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} xdx , \text{ and now dividing thru by } n \text{ gives:}
\]

\[
\int \cos^n dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} xdx
\]

b) let \( n = 2 \):

\[
\int \cos^2 xdx = \frac{1}{2} \cos x \sin x + \frac{1}{2} \int dx = \frac{x}{2} + \frac{\sin 2x}{4} + C \text{ now using the}
\]
c) $\int \cos^4 x \, dx = \frac{1}{4} \cos^3 x \sin x + \frac{3}{4} \int \cos^2 x \, dx$

now using the result in part (b), this gives:

$\int \cos^4 x \, dx = \frac{1}{4} \cos^3 x \sin x + \frac{3}{8} x + \frac{3}{16} \sin(2x) + C$