11-26

Note Title 11/26/2007

<u>Ex</u> US population in 1970 = 203 million

1980 = 227 million

Define $t = (\text{year}) - 1970$; P = millions of people.

$$P(t) = P_0 e^{kt} = 203\, e^{kt}$$

To find k, let $t = 10$. $227 = P(10) = 203\, e^{10k}$

$$e^{10k} = \frac{227}{203}, \quad 10k = \ln\left(\frac{227}{203}\right), \quad k = \frac{1}{10} \ln\left(\frac{227}{203}\right) \approx 0.01117 \text{ yr}^{-1}$$

Q1: Predicted population for 2000: $203\, e^{30k} \approx 283.8$ (Actual: 281)

Q2: When is $P = 300$? $300 = 203\, e^{kt} \Rightarrow kt = \ln\left(\frac{300}{203}\right) \Rightarrow t = 35.0$ yr (2005)

Ex If you earn continuous interest rate of 9% per year, and you want \$1,000,000 after 40 years, how much principal do you need to invest?

$$10^6 = M(40) = M_0 e^{40r} = M_0 e^{(0.09)(40)} = M_0 e^{3.6}$$

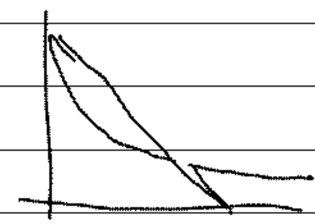
$$M_0 = \frac{10^6}{e^{3.6}} \approx 27,320$$

__Ex__ Polonium-210 has a half-life of 140 days.

$$kt_H = -\ln(2), \quad \text{so} \quad k = \frac{-\ln(2)}{140} = -0.004951 \ \text{day}^{-1}$$

Q: How long until 75% of original remains?

$$\frac{3}{4}\cancel{m_0} = \cancel{m_0}e^{kt} \qquad kt = \ln\left(\frac{3}{4}\right) \qquad t = \frac{\ln(3/4)}{k} = 58.1 \text{ days}$$



Q: How long until 25% remains? $\quad \frac{1}{4} = \frac{1}{2}\cdot\frac{1}{2}$

$$2t_H = 280 \text{ days}$$