10-24

Note Title 10/24/2007

Ex $0.9999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots = \sum_{n=1}^{\infty} \frac{9}{10^n}$

$$= 9 \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n = 9 \cdot \frac{1}{10} \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^{n-1}$$

$$= \frac{9}{10}\left(\frac{1}{1-\frac{1}{10}}\right)$$ geometric series

$$= \frac{9}{10}\left(\frac{10}{9}\right)$$

$$= 1$$

Ex $$\sum_{n=1}^{\infty} \frac{e^n}{(-2)^{n+1}} = -\frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{e}{-2}\right)^n = -\frac{1}{2} \cdot \left(-\frac{e}{2}\right) \sum_{n=1}^{\infty} \left(-\frac{e}{2}\right)^{n-1}$$

geometric series with $r = -\frac{e}{2}$

$|r| > 1 \implies$ diverges

Ex

$$\sum_{n=1}^{\infty}\left[(0.8)^{n-1} - (0.4)^n\right] = \sum_{n=1}^{\infty}(0.8)^{n-1} - \sum_{n=1}^{\infty}(0.4)^n \quad \text{if both converge}$$

$$= \frac{1}{1-\frac{4}{5}} - \frac{2}{5}\left(\frac{1}{1-\frac{2}{5}}\right) \quad \checkmark$$

$$= 5 - \frac{2}{\cancel{5}}\left(\frac{\cancel{5}}{3}\right) = \frac{13}{3}$$

Ex $\sum_{n=1}^{\infty} 1$ $a_n = 1$ for all n

$\{a_n\} \to 1$ not zero $\therefore$ series diverges

$\sum_{n=1}^{\infty} (-1)^{n-1}$ $a_n = (-1)^{n-1}$

$\{a_n\}$ does not converge $\therefore$ series diverges

$\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+3}\right)$ $a_n = \ln\left(\frac{n}{2n+3}\right)$

$\{a_n\} \to \ln\left(\frac{1}{2}\right) \neq 0$

$\therefore$ series diverges

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2+1}$ $\qquad a_n = \dfrac{1}{4n^2+1} \to 0$ $\qquad$ Divergence Test inconclusive

$f(x) = \dfrac{1}{4x^2+1}$

$a_n = f(n)$ ✓

$f(x)$ continuous ✓

$f(x)$ decreasing ✓ $\quad f'(x) = -(4x^2+1)^{-2}(8x)$

< 0 for $x \geq 1$

$f(x) > 0$ ✓

Apply Integral Test:

$$\int_1^\infty f(x)\,dx = \int_1^\infty \frac{1}{4x^2+1}\,dx = \int_2^\infty \frac{1}{u^2+1}\,\frac{1}{2}\,du = \frac{1}{2}\left[\tan^{-1}(u)\right]_2^\infty$$

$u = 2x$

$du = 2dx$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \tan^{-1}(2)\right] < \infty$$

$\Uparrow$

Series converges