

$$\underline{\text{Ex}} \int \sin^4 \theta \, d\theta$$

$$\begin{array}{l} m=4 \\ n=0 \end{array} \quad \text{both even}$$

$$= \int (\sin^2 \theta)^2 \, d\theta = \int \left[\frac{1}{2}(1 - \cos 2\theta) \right]^2 \, d\theta = \frac{1}{4} \int (1 - 2\cos 2\theta + \cos^2 2\theta) \, d\theta$$

$$= \frac{1}{4}\theta - \frac{1}{2} \left(\frac{1}{2} \sin 2\theta \right) + \frac{1}{4} \int \cos^2 2\theta \, d\theta$$

$m=0$ both even
 $n=2$

$$+ \frac{1}{4} \int \frac{1}{2}(1 + \cos 4\theta) \, d\theta$$

$$+ \frac{1}{8}\theta + \frac{1}{8} \left(\frac{1}{4} \sin 4\theta \right) + C$$

$$= \frac{3}{8}\theta - \frac{1}{4} \sin 2\theta + \frac{1}{32} \sin 4\theta + C$$

$\cos^2 2\theta = \frac{1}{2}(1 + \cos 4\theta)$

Sometimes you can do this with negative powers.

$$\int \frac{\sin^3 x}{\cos^6 x} dx = \int \frac{\sin^2 x}{\cos^6 x} \sin x dx = \int \frac{1 - \cos^2 x}{\cos^6 x} \sin x dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$= \int \frac{1 - u^2}{u^6} (-du) = \int \left(\frac{u^2}{u^6} - \frac{1}{u^6} \right) du = \frac{1}{-3} u^{-3} - \frac{1}{-5} u^{-5} + C$$

$$= \frac{1}{5} \cos^{-5} x - \frac{1}{3} \cos^{-3} x + C$$

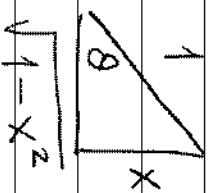
~~Ex~~

$$\int \frac{1}{(1-x^2)^{3/2}} dx \quad (1-x^2)^{1/2} : x = 1 \sin \theta$$

$$dx = \cos \theta d\theta$$

$$= \int \frac{1}{(1-\sin^2 \theta)^{3/2}} (\cos \theta d\theta) = \int \frac{1}{\cos^3 \theta} \cos \theta d\theta = \int \frac{d\theta}{\cos^2 \theta}$$

$$= \int \sec^2 \theta d\theta = \tan \theta + C$$



$$x = \sin \theta$$

$$= \frac{x}{\sqrt{1-x^2}} + C$$

Ex

$$\int \frac{dx}{a^2+x^2}$$

$$(a^2+x^2)^{1/2} : \quad x = a \tan \theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$$
$$dx = a \sec^2 \theta \, d\theta$$

$$= \int \frac{a \sec^2 \theta \, d\theta}{a^2(1+\tan^2 \theta)} = \int \frac{a \sec^2 \theta \, d\theta}{a^2 \sec^2 \theta} = \int \frac{d\theta}{a} = \frac{\theta}{a} + C$$
$$= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\text{Ex} \int_{2\sqrt{3}}^2 \frac{1}{x^2 \sqrt{x^2+4}} dx$$

$$\sqrt{x^2+4} : x = 2 \tan \theta$$

$$dx = 2 \sec^2 \theta d\theta$$

$$\text{If } x=2, 1 = \tan \theta \Rightarrow \theta = \frac{\pi}{4}$$

$$\text{If } x=2\sqrt{3}, \sqrt{3} = \tan \theta \Rightarrow \theta = \frac{\pi}{3}$$

$$\int_{\pi/4}^{\pi/3} \frac{1}{4 \tan^2 \theta \sqrt{4+4 \tan^2 \theta}} \cdot 2 \sec^2 \theta d\theta = \int_{\pi/4}^{\pi/3} \frac{\sec^2 \theta}{4 \tan^2 \theta \sec \theta} d\theta$$

$$= \int_{\pi/4}^{\pi/3} \frac{\cos^2 \theta}{4 \sin^2 \theta \cos \theta} d\theta = \frac{1}{4} \int_{\pi/4}^{\pi/3} \frac{\cos \theta}{\sin^2 \theta} d\theta = \frac{1}{4} \left[-\frac{1}{\sin \theta} \right]_{\pi/4}^{\pi/3}$$

$$= \frac{1}{4} \left[-\frac{2}{\sqrt{3}} + \sqrt{2} \right]$$