Numerical conformal mapping

2003 and beyond

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- Prelude
- Current technologies
- Recent developments and trends
- Applications

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Evolution

For a century after Riemann's mapping theorem in 1851, the flexibility and accuracy of constructive conformal maps was rather limited.

Now, however, an incredible variety of maps can be conveniently used. In fact, computation has become the primary driver of interest in conformal mapping.

The role of numerical CM

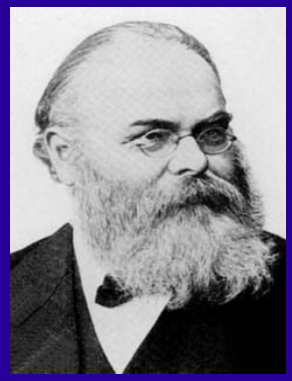
CM is computationally most useful when the difficult aspect of a problem lies in, or can be expressed soley in terms of,

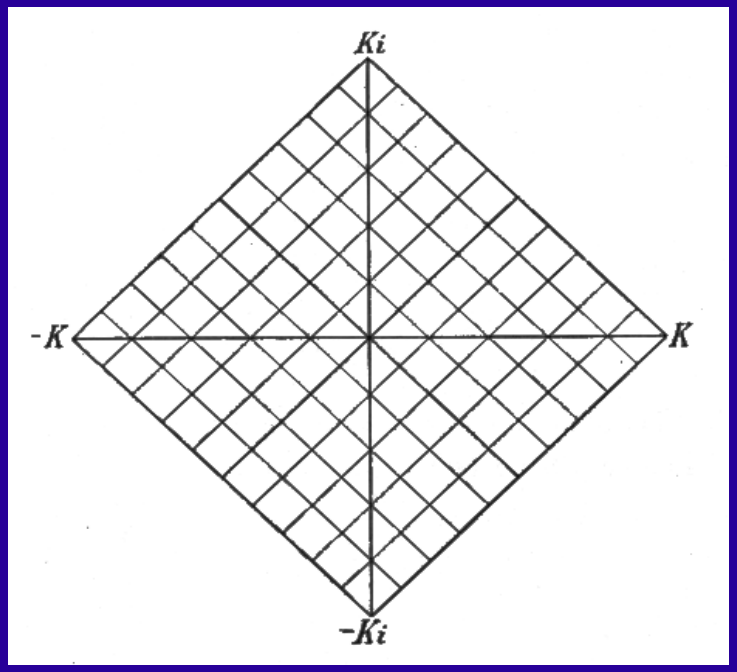
GEOMETRY

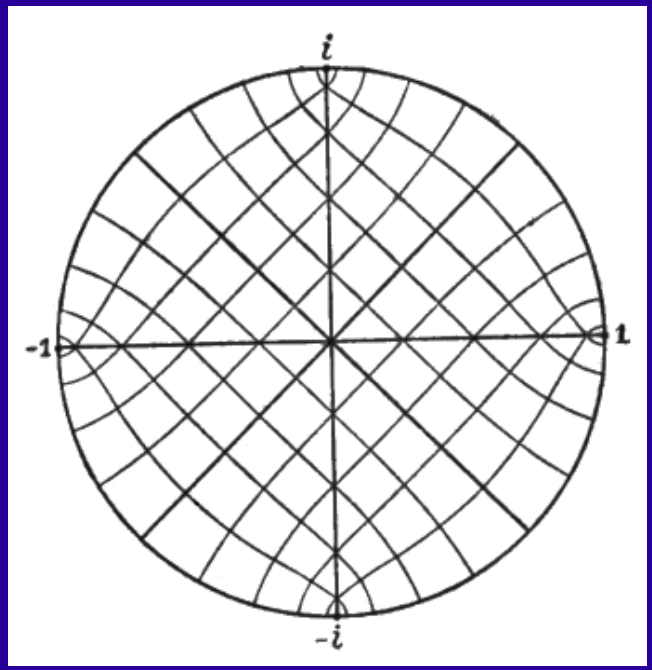
Of course, CM remains inherently interesting.

1869

Schwarz computes the map to a square and includes a plot in his landmark paper.



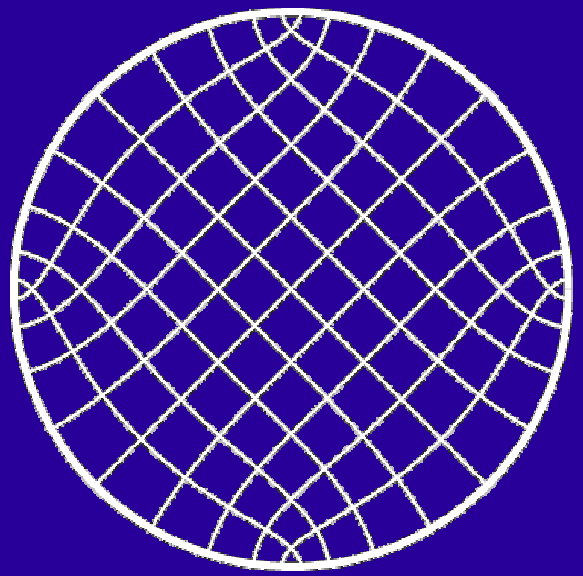




2001

Driscoll uses software to create the same plot.

```
p = polygon([1+i,-1+i,-1-i,1-i]);
f = diskmap(p);
f = center(f,0);
plot(exp(i*linspace(0,2*pi,180)));
hold on, axis equal
[X,Y] = meshgrid((-4:4)/5,...
    (-100:100)/100);
plot(evalinv(f,X+i*Y),'k')
[X,Y] = meshgrid((-100:100)/100,...
    (-4:4)/5);
plot(evalinv(f,X'+i*Y'),'k')
```



(And a few others.)

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SC Toolbox

Driscoll, 1994-2003

- Public domain package for MATLAB.
- Implements Schwarz-Christoffel maps to regions bounded by polygons.
- Graphical and command interfaces.
- Very accurate; reasonably fast for up to dozens of vertices.
- Includes fast solution of Laplace's equation with PC boundary conditions.

DSCPACK

Hu, 1995

- Public domain Fortran 77 package.
- Implements SC maps to doubly connected regions bounded by polygons.
- Accurate; fast for dozens of vertices.
- Now being adapted for use in the SC Toolbox (Heryudono).

CONFPACK

Hough, 1990

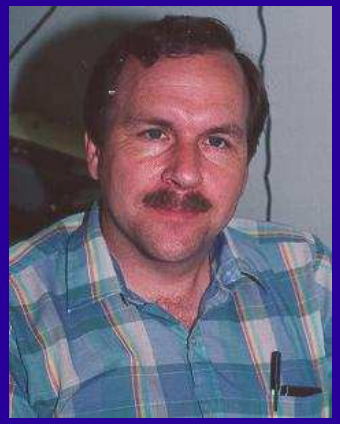
- Public domain Fortran 77 package.
- Implements Symm's integral equation for maps to simply connected regions with piecewise smooth boundary.
- Includes special treatment of corners.
- Fast and rather accurate for many regions.

ZIPPER

Marshall, 1996-97

- Public-domain C software.
- Implements a method of successive approximations for simply connected regions.
- Includes graphical output (in X11).
- Demos and applications included.

CirclePack



Stephenson, 1992-2003

- Public-domain C/xview package.
- Implements circle packings, which converge to conformal maps, for a very wide range of geometries.
- Extremely flexible and adaptable approach.
- Fast for modest accuracy.

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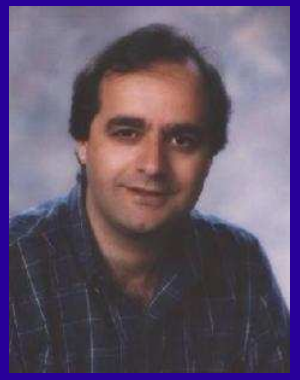
Domain decomposition

- Symm's equation, for improved efficiency (Driscoll)
- S-C formula, for arbitrarily elongated regions without crowding (D & Vavasis)
- Patching together simple maps, to build complicated regions (Stylianopoulous, Papamichael, and Falcao)



Multipole evaluation

- Use multipole evaluation for an integral kernel (O'Donnel and Rokhlin) or the S-C integrand (Banjai and Trefethen).
- Work grows linearly with the size of the discretization.
- Enables S-C maps to fractal or smooth boundaries.

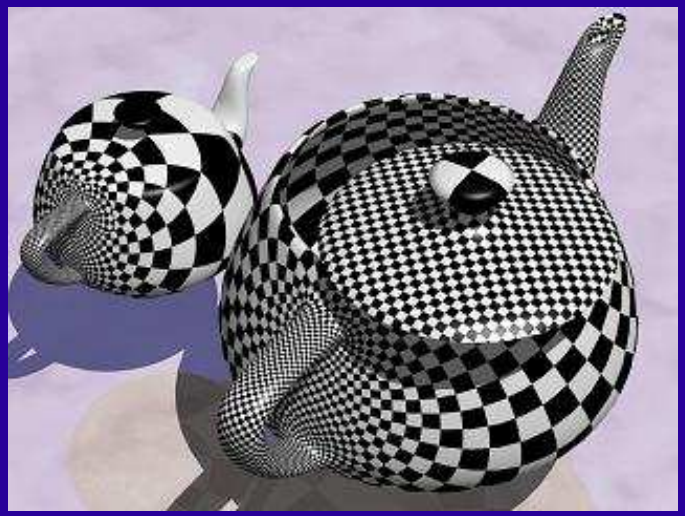
Multiply connected S-C

- S-C formula can be extended to polygonally bounded regions of any connectivity (DeLillo, Elcrat, and Pfaltzgraff).
- Use method of images to create S-C singularities.
- Computationally feasible...but could multipole ideas help?

Nonplanar surfaces

Methods for surfaces embedded in space:

- Circle packing (Collins & Stephenson)
- S-C (Collins, Stephenson, & D)
- Laplace-Beltrami (Gu & Yau, others)



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Laplace's equation

Solving Laplace in 2D is no longer a big deal, even with geometry, unless you require

- high accuracy
- careful corner treatment
- semi-analytical information

(S-C maps are good at all three!)

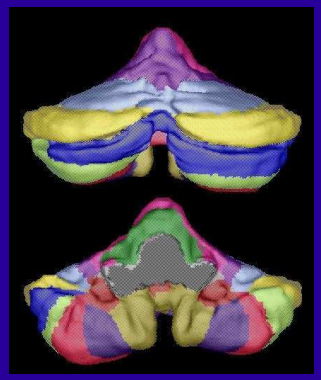
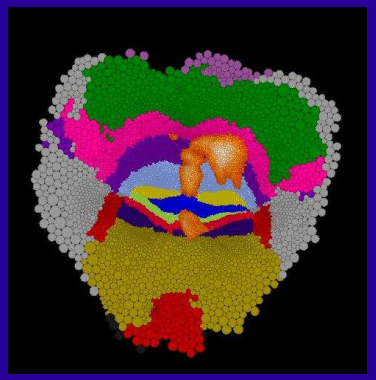
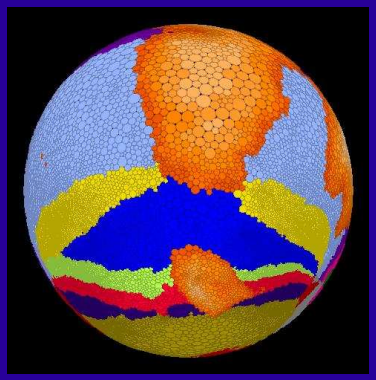
Such applications seem to come most often now from electrical and electronic devices.

Results in analysis

- Omitted-area problem (Banjai & Trefethen)
- Convergence of Krylov subspace methods (Embree et al.)
- Sullivan's constant, Brennan's conjecture? (Epstein et al.)
- Discrete conformal maps/circle packing (Stephenson)

Imaging

- Computer graphics: accurate normals (Gu et al)
- Medical imaging/brain maps (Hurdal)

- Face scans?