Exercise 1: Multivariate normal distribution

1. Load the `mvtnorm` library.
2. Construct a mean vector \( \mu = (0, 0) \) and a covariance matrix \( \Sigma = X^T X \) for some random matrix \( X \in \mathbb{R}^{2 \times 2} \).
3. Construct vectors \( x, y \) and a matrix \( z \) as follows:
   
   ```r
   x = seq(-1,1,by=0.05)
   y = seq(-1,1,by=0.05)
   n = length(x)
   z = matrix(0,nrow=n, ncol=n)
   ```

4. Compute \( z[i,j] = f(x[i], y[j]) \) where \( f \) is the density of \( N(\mu, \Sigma) \). (Use the `dmvnorm` command).
5. Make a contour plot of the normal density: `contour(x,y,z)`.
6. Add the eigenvectors of \( \Sigma \) to the plot
   
   ```r
   e = eigen(Sigma)
   arrows(0,0,e$vectors[1,1], e$vectors[2,1])
   arrows(0,0,e$vectors[1,2], e$vectors[2,2])
   ```

7. Density in 3d: `persp(x,y,z,theta = 30, phi = 30)`.
1. Load the Titanic training dataset (available on Sakai).
2. Split the data into a training set (2/3) and a test set (1/3).
3. Run `contrasts(data$Embarked)` to see how R handles categorical variables such as Embarked.
4. Train a logistic regression model to try to predict the fate of the passengers using some of the features:
   ```r
   model = glm(Survived ~ x1 + x2 + ... ,
               family=binomial(link='logit'),data=train)
   ```
5. Predict values on the test set:
   ```r
   yhat = predict(model, test, type='response')
   ```
   (Note: returned values are of the form $P(Y = 1|X = x)$ because of the “response” option).
6. Compute the prediction accuracy of your model.