1. Load the Auto dataset.

2. Use the `lm` function to fit a linear model
   \[ mpg = \beta_0 + \beta_1 \cdot \text{horsepower} + \beta_2 \cdot \text{weight}. \]

3. Compute the coefficients directly by solving the normal equations. Do you get the same results?

Note: You may need to convert the data frame to a matrix using `as.matrix(X)`. 
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Note: You may need to convert the data frame to a matrix using `as.matrix(X)`.

If you do not get the same results: did you include an intercept in the normal equations?

```r
X = as.matrix(Auto[,c(4,5)])
Xp = cbind(matrix(1,392,1), X)
```
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- A complex model that fits data very well will often make poor predictions. **Overfitting.**
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3. We use the fitted model to predict values of the test data and compute the **test error**.
Splitting data into training/test data:

In the case of least squares:

\[
\hat{\beta} = \left( X^T \text{train} X \text{train} \right)^{-1} X^T \text{train} Y \text{train}.
\]

\[
\hat{Y} \text{test} = X \text{test} \hat{\beta}.
\]

Test error:

\[
\text{MSE test} = \frac{1}{n^2} \sum_{i=1}^{n2} \left( \hat{Y} \text{test}_i - Y \text{test}_i \right)^2.
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We choose a model that minimizes the test error.
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Training error and test error (cont.)

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We choose a model that minimizes the test error.
Typical behavior of the test and training error, as model complexity is varied.
Train/test sets in R

```r
library(ISLR)
data(Auto)

Auto <- Auto[, -9]  # Remove the "names" column

n <- dim(Auto)[1]
ntrain <- floor(0.75 * n)
ntest <- n - ntrain

train_ind <- sample(1:n, ntrain)

train <- Auto[train_ind,]

test <- Auto[-train_ind,]
```

Compute the test error:

```r
model_full <- lm(mpg ~ ., data=train)
mean((predict(model_full, test[, -1]) - test[, 1])^2)
```

6/11
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Using a subset of variables

Fit a model using only the last 3 variables:

```r
model <- lm(mpg ~ ., data=train[, append(c(5,7,8),1)])
mean((predict(model, test[, c(5,7,8)]) - test[,1])**2)
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model <- lm(mpg ~ ., data=train[,append(c(5,7,8),1)])
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```

Minimal test error for subsets of a given size:
Examining all subsets

For this dataset, we can examine all the possible subsets (usually impossible):
Best subset selection: Given $k \in \{1, \ldots, p\}$, we find the subset of size $k$ of $\{1, \ldots, p\}$ that minimizes the prediction error.
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- Note: there are $\binom{p}{k}$ subsets of size $k$ and $2^k$ possible subsets. So the procedure is only computationally feasible for small values of $p$.

- The leaps and bounds procedure (Furnival and Wilson, 1974) makes this feasible for $p$ as large as 30 or 40.
Forward- and Backward- stepwise regression

- Best subset selection performs well, but is too computationally intensive to be useful in practice.

Two natural greedy variants of the best subset selection technique:

**Forward stepwise regression:** starts with the intercept, and then sequentially adds into the model the predictor that most improves the $t$.

**Backward stepwise regression:** starts with the full model, and sequentially deletes the predictor that has the least impact on the $t$.

Can be used even when the number of variables is very large.

However, greedy approach: doesn’t guarantee a global optimum. Less rigorous than other methods, less supporting theory.

Nevertheless, the stepwise approaches often return predictors similar to the predictors obtained from more complex methods with better theory.
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Nevertheless, the stepwise approaches often return predictors similar to the predictors obtained from more complex methods with better theory.
1 Install and load the leaps package.

2 Use the regsubsets function to perform forward and backward stepwise regressions.

```R
library(leaps)

regfit.fwd = regsubsets(mpg ~ ., data=Auto[,,-9], method="forward")

regfit.bwd = regsubsets(mpg ~ ., data=Auto[,,-9], method="backward")
```

3 Examine the output of summary(regfit.fwd) and plot(regfit.fwd).