MATH201
Summer College 2003

Quiz 4

Name: Key 50

Instructions:

1. Do not start until instructed to do so.
2. You may use a calculator and one 3" x 5" card (front and back) with notes, but nothing else.
3. The work you turn in must be your own.
4. SHOW ALL WORK.
Questions 1,2: One of the questions asked in a recent Gallup poll (April 5-6, 2003) was, "How would you say the war with Iraq has gone for the U.S. so far (very well, moderately well, moderately badly, or badly)?" Five hundred forty out of 1,058 adult respondents aged 18 years or older responded "very well." According to Gallup, for results based on this sample, one can say with 95% confidence that the error attributable to sampling and other random effects is ± 3 percentage points.

1. **4 points** Assuming that this sample represents the U.S. adult population, can you say that a majority of the U.S. adult population thought the war had gone "very well" for the U.S. at that time? Why or why not?

\[ \hat{p} \pm \text{margin of error} = \frac{540}{1058} \pm 0.03 \rightarrow 0.51 \pm 0.03 \rightarrow (0.48, 0.54) \]

No. The 95% confidence interval for the proportion of all adults who would respond "very well" contains 0.5.

2. **4 points** Using the same level of confidence, how many people would Gallup need to survey in order to estimate the proportion who think the war has gone "very well" for the U.S. with a margin of error of only 2 percentage points?

\[ z_{a/2} = 1.96 \quad \hat{p} = 0.51 \quad \beta = 0.02 \]

\[ n = \frac{z_{a/2}^2 \hat{p} (1-\hat{p})}{\beta^2} = \frac{1.96^2 (0.51 \times 0.49)}{0.02^2} = 2400.034 \rightarrow 2401 \]

Questions 3-4: Runzheimer International publishes results of studies on overseas business travel costs. Suppose as a part of one of these studies the per diem travel expense amounts (in dollars) are obtained for 22 business travelers staying in Johannesburg, South Africa. Summaries are shown below. We would like to use these data to estimate the average per diem travel expense for all businesspeople traveling to Johannesburg. Assume that expense amounts are normally distributed.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>TrMean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>expense</td>
<td>22</td>
<td>152.16</td>
<td>150.02</td>
<td>151.89</td>
<td>14.42</td>
<td>3.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>expense</td>
<td>129.28</td>
<td>178.34</td>
<td>142.35</td>
<td>161.94</td>
</tr>
</tbody>
</table>

3. **1 point** Give the point estimate.

\[ \bar{x} = 152.16 \]

4. **5 points** Construct and interpret a 99% confidence interval for the population average per diem travel expense.

\[ \text{Use } t. \quad df = 21 \quad t_{a/2} = t_{0.005} = 2.831 \]

\[ \bar{x} \pm t_{a/2} \frac{s}{\sqrt{n}} \rightarrow 152.16 \pm 2.831 \left( \frac{14.42}{\sqrt{22}} \right) \rightarrow (143.44, 160.86) \]

We are 99% confident that the avg. per diem travel expense for all business travelers to Johannesburg is between $143.44 and $160.86.
5. 3 points  In our class survey, we collected data on the number of CDs owned by each student. Summaries are shown below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>TrMean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDs</td>
<td>26</td>
<td>80.0</td>
<td>50.0</td>
<td>69.9</td>
<td>88.2</td>
<td>17.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDs</td>
<td>3.0</td>
<td>400.0</td>
<td>20.0</td>
<td>105.0</td>
</tr>
</tbody>
</table>

If you wanted an interval estimate of the number of CDs owned by the average high school senior, which confidence interval tool would be appropriate?

A. $\bar{x} \pm z_{a/2} \frac{\sigma}{\sqrt{n}}$

B. $\bar{x} \pm z_{a/2} \frac{s}{\sqrt{n}}$

C. $\bar{x} \pm t_{a/2} \frac{\sigma}{\sqrt{n}}$

D. $\bar{x} \pm t_{a/2} \frac{s}{\sqrt{n}}$

E. Sign interval

6. 4 points A researcher wishes to estimate the mean number of miles on all four-year-old Saturn SC1s. How many cars should be in a sample in order to estimate the mean number of miles to within 1000 miles with 92% confidence, assuming that $\sigma = 19,700$?

$$n = \left( \frac{z_{a/2} \cdot \sigma}{B} \right)^2 = \left( \frac{1.75 \cdot 19,700}{1000} \right)^2$$

$$= 1188.52 \rightarrow 1189$$

Questions 7-9. A math teacher claims that she has developed a review course that increases the scores of students on the math portion of the SAT exam. Based on data from the College Board, SAT scores are normally distributed with mean = 514 and standard deviation = 113. The teacher obtains a random sample of 3600 students, puts them through the review class, and finds that the mean SAT math score of the 3600 students is 518.

7. 3 points Explain what a Type II error would be in terms of this problem.

Failing to find evidence that the review course increases scores, on average, when it really does.
8. 12 points  Is this enough evidence that the mean SAT math score for all students who take the review class is higher than those in general? Use $\alpha = .10$. Set up the hypotheses, give the test statistic, critical value(s), and p-value, and make a conclusion in terms of the problem.

$H_0: \mu = 514$

$H_1: \mu > 514$

$$Z_{TS} = \frac{518 - 514}{113/\sqrt{3000}} = 2.12$$

$p$-value $< \alpha$  \[ \rightarrow \text{ reject } H_0 \]

We have enough evidence to conclude that the mean SAT math score for all students who take the review course is higher than those in general.

9. 4 points  Give the p-value for your hypothesis test above if $n = 40$. Keep everything else the same. What does this tell you about how sample size is related to the amount of evidence in a sample against the null hypothesis?

$$Z_{TS} = \frac{518 - 514}{113/\sqrt{40}} = .22$$

$$p$-value = .5 - .0871 = .4129$

The smaller the sample size, the less evidence against $H_0$. 
10. **8 points** An engineer suspects that a certain pH meter is biased toward acidic. She uses the meter to measure the pH in 14 neutral substances (pH = 7.0) and obtains the data below. Is there sufficient evidence that the pH meter is giving readings that are too low, on average? Use $\alpha = 0.02$.

Set up the hypotheses, give the test statistic, critical value(s) or the p-value, and answer the question with either "yes" or "no."

<table>
<thead>
<tr>
<th>pH</th>
<th>Mean</th>
<th>StdDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.01</td>
<td>7.04</td>
<td>6.97</td>
<td>7.00</td>
</tr>
<tr>
<td>7.04</td>
<td>7.01</td>
<td>7.00</td>
<td>6.99</td>
</tr>
</tbody>
</table>

Variable | N     | Mean   | StdDev  | SE Mean |
---------|-------|--------|---------|---------|
PH       | 14    | 7.00786| 0.03043| 0.00813 |

H₀: $\mu = 7.0$
Hₐ: $\mu < 7.0$

Use $t$ test

$$t_{TS} = \frac{7.00786 - 7.0}{0.03043/\sqrt{14}} = 9.106$$

$df = 13$

**Crit. Value**

$0.025 \rightarrow t_{TS} = 2.160$

**p-value**

$0.50 < p-value < 0.90$

$t_{TS}$ not in CR $\rightarrow$ do not reject $H₀$

$p-value > \alpha$ $\rightarrow$ reject $H₀$

$\bigcirc \no$