MATH201
Summer College 2003

Quiz 3

Name: Key / 50

Instructions:

1. Do not start until instructed to do so.
2. You may use a calculator and one 3” x 5” card (front and back) with notes, but nothing else.
3. The work you turn in must be your own.
4. SHOW ALL WORK.
Questions 1-5: The Environmental Protection Agency publishes data regarding the gas mileage of all cars. Suppose that the average highway gas mileage for all large cars manufactured in 1999 is 25.1 miles per gallon and the standard deviation is 3.9 miles per gallon. Assume that the population of gas mileages is normally distributed.

1. **5 points** You are shopping for a large car made in 1999 but cannot afford to buy one that gets poor gas mileage considering the current price of gasoline. You would like your car to get at least 30 miles per gallon. What proportion of large cars made in 1999 meet this criteria?

\[ Z = \frac{30 - 25.1}{3.9} = 1.26 \]

\[ P(Z > 1.26) = 0.1038 \]

2. **5 points** Fill in the blank: 90% of all large cars made in 1999 have gas mileages above ___ mpg.

\[ Z = -1.28 \]

\[ -1.28 = \frac{x - 25.1}{3.9} \]

\[ x = 20.108 \]

3. **5 points** You decide to test 49 large cars made in 1999 to observe their gas mileage. What is the probability that the average gas mileage for these 49 cars will be within 1 mpg of the population mean?

\[ \mu_x = \mu = 25.1 \]

\[ \sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{3.9}{\sqrt{49}} \]

\[ \bar{x} \text{ is normal by CLT} \]

\[ Z = \frac{25.1 - 25.1}{3.9/7} = 1.79 \]

\[ Z(1.79) = 0.9266 \]

4. **3 points** True or False: Your answer to 3 would change if the distribution of gas mileages is not normal.
5. **8 points** A researcher claims that fuel additives increase the gas mileage, on average, of cars driven under highway driving conditions. The researcher obtains a random sample of 35 large cars manufactured in 1999 and adds a fuel additive before measuring gas mileage. With the additive the sample mean gas mileage for these 35 cars is 26.8 miles per gallon. Assume that the distribution of gas mileages with the additive has a standard deviation of 3.9 miles per gallon. Can the researcher say, with 94% confidence, that the fuel additive is effective in increasing highway gas mileage, on average? Explain.

\[ n = 35 \quad \bar{x} = 26.8 \quad \sigma = 3.9 \]

94% confidence \[ \alpha = 0.06 \] \[ z_{0.06} = 1.88 \]

\[ \bar{x} \pm z_{0.06} \frac{\sigma}{\sqrt{n}} \rightarrow (25.56, 28.04) \]

Yes. This confidence interval puts the population mean at least at 25.56 which is higher than 25.1.

6. **4 points** If \( Z \) is a standard normal random variable, find \( P(Z > -0.50) \).

\[ .5 + .1915 = .6915 \]

7. **4 points** If \( Z \) is a standard normal random variable, find \( z_0 \) such that \( P(Z < z_0) = .0040 \).

\[ z_0 = -2.65 \]

Questions 8-11: A Gallup poll conducted December 20-21, 1999 asked a random sample of Americans, "How often do you bathe each week?" The sample average response was 6.9 times and Gallup determined that the margin of error corresponding to 95% confidence was 0.17. A similar study conducted June 29 – July 4, 1950 gave an average response of 3.7 times with a margin of error of 0.14.

8. **5 points** Estimate the average number of times Americans bathed per week in 1999 using a 95% confidence interval. Interpret your interval.

\[ \bar{x} \pm \text{margin of error} \rightarrow 6.9 \pm 0.17 \rightarrow (6.73, 7.07) \]

We are 95% confident that the average number of times Americans bathed per week in 1999 is between 6.73 and 7.07.

9. **3 points** How could you make your confidence interval in 8 more precise (narrower)?

increase the sample size or decrease the confidence level
10. **5 points** Estimate the average number of times Americans bathed per week in 1950 using a 95% confidence interval. Interpret your interval.

\[ \bar{x} \pm \text{margin of error} \rightarrow 3.7 \pm .14 \rightarrow (3.56, 3.84) \]

We are 95% confident that the average number of times Americans bathed per week in 1950 is between 3.56 and 3.84.

11. **3 points** Is there compelling evidence that the average American in 1999 bathed more frequently than in 1950? Explain.

Yes! These two confidence intervals do not overlap, so, worst case scenario, if \( m_{1999} \) is 6.73 and \( m_{1950} \) is 3.84, there's still quite a difference... at least with 95% confidence.