**INTRODUCTION**

An expander graph is a sparse, but highly connected graph. More precisely, each node has a constant number of neighbors and in order to disconnect an expander graph into two parts, one has to remove many edges (proportional to the number of nodes in the smaller part). Because of these nice properties, expander graphs have been applied in complexity theory, design of robust computer networks, and the theory of error-correcting codes. In our research, we try to construct expander graphs by using signed adjacency matrices.

- A **graph** is a pair $(V, E)$ of sets comprising a set $V$ of **vertices** together with a set $E$ of **edges**, which are 2-element subsets of $V$.

- A **d-regular graph** is **Ramanujan** if all of its non-trivial eigenvalues lie between $-2\sqrt{d-1}$ and $2\sqrt{d-1}$.

- The **adjacency matrix** of a graph $G$ on $n$ vertices is an $n \times n$ $\{0,1\}$-matrix for which entry $[i,j]$ has a $1$ if vertices $i$ and $j$ are adjacent and $0$ otherwise.

**SIGNING & 2-lifts**

We define a **signing** of a graph $G=(V,E)$ as a function $\sigma : V \rightarrow \{0,1\}$, together with the adjacency matrix $A^\sigma$ of $G$ together with the eigenvalues of $A^\sigma$, including multiplicity.

Calculating the eigenvalues of $A$, $A^\sigma$ and $\tilde{A}^\sigma$ we see the corresponding spectra are $\{2,-1,-1\}$, $\{2,-1,-1\}$ and $\{-2,1,1\}$, respectively.

**THEOREM 1** (Marcus, Spielman, Srivastava 2013) If $G$ is a $d$-regular graph, there is a signing so that the eigenvalues of $A^\sigma$ are at most $2\sqrt{d-1}$.

**FINDING SIGNING WITH THE SMALLEST SPECTRAL RADIUS**

Every graph $G$ has a signing in which all of the new eigenvalues are less than the spectral radius of its universal cover. Applying the 2-lifts of these signing inductively to any finite regular bipartite Ramanujan graph yields a family of regular bipartite Ramanujan graphs whose degree distribution matches that of the origin graph. We have focused on finding the signing with the smallest spectral radius for cubic graphs this summer, the following are examples of 10 and 12 vertices, respectively.

**REFERENCES**


**PREFERRIMARIES**

**CONJECTURE 1** (Bilu-Linial 2006) Every $d$-regular graph has a signing with spectral radius at most $2\sqrt{d-1}$.

**THEOREM 1** (Marcus, Spielman, Srivastava 2013) If $G$ is a $d$-regular graph, there is a signing so that the eigenvalues of $A^\sigma$ are at most $2\sqrt{d-1}$.

**THEOREM 2** (Marcus, Spielman, Srivastava 2013) For every $d \geq 3$ there is an infinite sequence of $d$-regular bipartite Ramanujan graphs.

**THEOREM 1** is a weak version of **CONJECTURE 1**. The difference between them is that **THEOREM 1** does not control the smallest eigenvalue of the signing. The eigenvalues of the adjacency matrices of bipartite graphs are symmetric about zero. So, a bound on the smallest non-trivial eigenvalue follows from a bound on the largest. This is why **THEOREM 1** is sufficient for proving the main result when we consider working with bipartite Ramanujan Graphs.

**PROOF OF EXISTENCE**

- **THEOREM 2** (Marcus, Spielman, Srivastava 2013) For every $d \geq 3$ there is an infinite sequence of $d$-regular bipartite Ramanujan graphs.

**Proof outline.**

- We use the fact that the complete $d$-regular bipartite graph is Ramanujan.
- Apply **LEMA 1** and **THEOREM 1**.
- This tells us that there is a 2-lift in which every non-trivial eigenvalues is less than $2\sqrt{d-1}$.
- Thus, for every $d$-regular bipartite Ramanujan graph $G$, there is another $d$-regular bipartite Ramanujan graph with twice as many vertices.

**FUTURE RESEARCH**

- Optimize the codes.
- Construct non-bipartite families of Ramanujan graphs.
- Is there a good way to find the 2-lifts that will work?