Math 351 Homework Set 8, Due Monday, December 1.

1. Find the eigenvalues and the corresponding eigenspaces of the following symmetric matrices

(a) $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$ (b) $\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$ (c) $\mathbf{A} = \begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 4 \\ 0 & 4 & 2 \end{pmatrix}$

(d) $\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ (e) $\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

(f) $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

2. For each part in the problem 1, use the eigenfuctions of the matrix to construct an orthogonal matrix Q. Then compute Q^{-1}. If there are multiple eigenvalues use the Gram- Schmidt process to obtain orthogonal vectors.

3. (a) Check whether the given vectors form an orthogonal set

$$\{[1\ \ 1\ \ 0]\,,\ [1\ \ -1\ \ 2]\}\,.$$

Then find all vectors orthogonal to the both above vectors.

(b) The same question as part (a) for the following set for

$$\{[-1\ \ 1\ \ -2\ \ 1],\ [3\ \ 1\ \ -1\ \ 0]\}$$

(c) Use part (a) to find an orthogonal basis for $\mathbb{R}^3$ that contains the vectors

$$[1\ \ 1\ \ 0] \qquad \text{and} \qquad [1\ \ -1\ \ 2]\,.$$

(d) Use part (b) to find an orthogonal basis for $\mathbb{R}^4$ that contains the vectors $[-1\ \ 1\ \ -2\ \ 1]$ and $[3\ \ 1\ \ -1\ \ 0]$.

4. Given the basis for $\mathbb{R}^3$

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}^\top, \ \mathbf{x}_2 = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}^\top, \ \mathbf{x}_3 = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}^\top$$

(a) Apply the Gram-Schmidt Process to obtain an orthogonal basis for the given basis.

(b) Normalize the basis found in (a) to obtain an orthonormal basis.

(c) Express the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^\top$ as a linear combination of the basis vectors found in part (b).